LIBERTAS MATHEMATICA, vol XXIV (2004)

Dynamical Hamiltonian Systems on Hamilton Spaces $H^{(k)n}$

Florian MUNTEANU

Abstract. In this paper we study dynamical Hamiltonian systems on some submanifolds of the dual bundle of a k-tangent bundle. Using the notions of symmetry and pseudosymmetry we obtain a kind of conservation laws for the higher order differentiable Hamiltonians, like in the case of classical Hamiltonians on T^*M (see [3], [13], [14], [16]). The particular case of Cartan spaces of higher order is also treated. Finally, on presented the cases of generalized Hamilton spaces and generalized Cartan spaces of order $k \geq 1$, are discussed.

Mathematics Subject Classification 2000: 37C10, 37K05, 53C60, 53D05, 70H33.

Keywords: conservation law, dynamical Hamiltonian system, Hamilton space of order $k \geq 1$, Cartan space of order $k \geq 1$, generalized Hamilton space of order $k \geq 1$, generalized Cartan space of order $k \geq 1$, Hamilton-Jacobi equations.

1 Introduction

Let M be a smooth, n-dimensional manifold, $C^\infty(M)$ the ring of real-valued smooth functions, $\mathcal{X}(M)$ the Lie algebra of vector fields and $A^p(M)$ the $C^\infty(M)$-module of p-differential forms, $1 \leq p \leq n$. For $X \in \mathcal{X}(M)$ with local expression $X = X^i(x)\frac{\partial}{\partial x^i}$ we consider the system of ordinary differential equations which give the flow $\{\Phi_t\}_t$ of X, locally,

$$\dot{x}^i(t) = \frac{dx^i}{dt}(t) = X^i(x^1(t), \ldots, x^n(t)), \quad i = 1, \ldots, n. \tag{1.1}$$

A dynamical system is a pair (M, X), where M is a smooth manifold and $X \in \mathcal{X}(M)$. A dynamical system is denoted by the flow of X, $\{\Phi_t\}_t$ or by the system of differential equations (1.1).

A function $f \in C^\infty(M)$ is called *a conservation law* for dynamical system (M, X) if f is constant along the all integral curves of X (solutions of (1.1)), that is

$$L_X f = 0, \tag{1.2}$$

where $L_X f$ means the Lie derivative of f with respect to X.

If $Z \in \mathcal{X}(M)$ is fixed, then $Y \in \mathcal{X}(M)$ is called *Z-pseudosymmetry* for (M, X) if there exists $f \in C^\infty(M)$ such that $L_X Y = fZ$. A X-pseudosymmetry for X is called *pseudosymmetry* for (M, X). $Y \in \mathcal{X}(M)$ is called *symmetry* for (M, X) if $L_X Y = 0$.

$\omega \in A^p(M)$ is called *invariant form* for (M, X) if $L_X \omega = 0$.

Now, if (M, ω) is a symplectic manifold ($\dim M = n = 2m$) then the dynamical system (M, X) is said to be *a dynamical Hamiltonian system* (or, shortly, *Hamiltonian system*) if there exists a function $H \in C^\infty(M)$ (called *the Hamiltonian*) such that

$$i_X \omega = -dH, \tag{1.3}$$

where i_X denotes the interior product with respect to X.

It is known that the symplectic form ω is an invariant 2-form for (M, X) and the divergence of the Hamiltonian vector field X vanishes.

From (1.3) it results that the Hamiltonian H is a conservation law for (M, X).

The next result which gives the association between pseudosymmetries and conservation laws is due to M. Crâşmăreanu ([3]) and G. L. Jones ([5]).

Proposition 1.1. *Let $X \in \mathcal{X}(M)$ be a fixed vector field and $\omega \in A^p(M)$ be a invariant p-form for X. If $Y \in \mathcal{X}(M)$ is symmetry for X and $S_1, \ldots, S_{p-1} \in \mathcal{X}(M)$ are $(p-1)$ Y-pseudosymmetry for X then*

$$\Phi = \omega(X, S_1, \ldots, S_{p-1}) \tag{1.4}$$

is a conservation laws for (M, X).

Particularly, if Y, S_1, ..., S_{p-1} are symmetries for X then Φ given by (1.4) is a conservation law for (M, X).

2 The dual bundle of k-tangent bundle

Let M be a real C^∞-manifold, n-dimensional and (T^kM, π^k, M) its k-accelerations bundle ($k \in \mathbb{N}^*$), which is also called *k-tangent bundle.* It can be identified with k-osculator bundle $(Osc^k M, \pi^k, M)$. A point $u \in T^kM$ has the coordinates $(x, y^{(1)}, \ldots, y^{(k)})$, $x \in M$ and $y^{(1)}$, ..., $y^{(k)}$ are the "higher order accelerations" ($y^{(h)} = \frac{1}{h!}\frac{d^h x}{dt^h}$, $h = \overline{1, k}$). The local coordinates of u are $(x^i, y^{(1)i}, \ldots, y^{(k)i})$.

We define *the dual* of (T^kM, π^k, M) as being $(T^{*k}M, \pi^{*k}, M)$, where $T^{*k}M$ is the following fibered product

$$T^{*k}M = T^{k-1}M \times_M T^*M. \tag{2.1}$$

Clearly, $(T^{k-1}M, \pi^{k-1}, M)$ is the $(k-1)$-accelerations bundle and (T^*M, π^*, M) is the cotangent bundle of the basic manifold M. So, $T^{*k}M$ is a C^∞-differentiable manifold and $\dim T^{*k}M = \dim T^kM = (k+1)n$. A point $u \in T^{*k}M$ is of the form $u = (x, y^{(1)}, \ldots, y^{(k-1)}, p)$, $\pi^{*k}(u) = x$ and u has the local coordinates $(x^i, y^{(1)i}, \ldots, y^{(k-1)i}, p_i)$.

For $k = 1$, $T^{*1}M$ is identified with T^*M. For more details, see the papers [10], [11] and the books [7], [9], [12] of Acad. Radu Miron, where all this mathematical objects were introduced.

Let us introduce the following differential forms ([9], [12]):

$$\begin{aligned} \theta &= p_i dx^i, \\ \omega &= d\theta = dp_i \wedge dx^i. \end{aligned} \tag{2.2}$$

The forms θ and ω are globally defined on the manifold $T^{*k}M$ and ω is *a canonical presymplectic structure* of rank $2n$ on the manifold $T^{*k}M$, $k > 1$.

On the manifold $T^{*k}M$ there exists at least a Poisson structure ([12]):

$$\{,\}_{k-1} : (f,g) \in C^\infty(T^{*k}M) \times C^\infty(T^{*k}M) \to \{f,g\}_{k-1} \in C^\infty(T^{*k}M)\,, \qquad (2.3)$$

defined by

$$\{f,g\}_{k-1} = \frac{\partial f}{\partial y^{(k-1)i}} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial y^{(k-1)i}}\,. \qquad (2.4)$$

3 Dynamical Hamiltonian systems on Hamilton spaces of order $k \geq 1$

A mapping $H : T^{*k}M \to \mathbb{R}$ is called *a differentiable Hamiltonian of order* k, $k \geq 1$, if H is a C^∞-function on $\widetilde{T^{*k}M} = T^{*k}M \setminus \{0\}$ and continuous on the null section of π^{*k}.

For a differentiable Hamiltonian $H(x, y^{(1)}, \ldots, y^{(k-1)}, p)$ we consider its Hessian with respect to p_i. Its matrix has the elements

$$g^{ij}(x, y^{(1)}, \ldots, y^{(k-1)}, p) = \frac{1}{2} \frac{\partial^2 H}{\partial p_i \partial p_j}. \qquad (3.1)$$

and g^{ij} is a d-tensor field on $T^{*k}M$, of type $(2,0)$, symmetric and contravariant. We say that H is *regular* if

$$rank\, ||g^{ij}|| = n \ \text{ on } \ \widetilde{T^{*k}M}. \qquad (3.2)$$

Definition 3.1. (R. Miron, [10], [11], [12]) *An Hamiltonian space of order k, $k \geq 1$, is a pair $H^{(k)n} = (M, H(x, y^{(1)}, \ldots, y^{(k-1)}, p))$ formed by a C^∞-manifold M, n-dimensional and a regular Hamiltonian of order k, H with the property that the d-tensor field g^{ij} has a constant signature on $\widetilde{T^{*k}M}$.*

As usually, H is called *the fundamental function* and g^{ij} *fundamental tensor field* of the Hamilton space of order k.

Now, let us consider the bundle $(T^{*k}M, \overline{\pi}^*, T^*M)$, where $\overline{\pi}^*(x, y^{(1)}, \ldots, y^{(k-1)}, p) = (x, p)$, and its canonical section

$$\sigma_0 : (x, p) \in T^*M \to (x, 0, \ldots, 0, p) \in T^{*k}M.$$

If we denote by $\Sigma_0 = \operatorname{Im} \sigma_0$ then it follows that Σ_0 is an immersed submanifold of the manifold $T^{*k}M$ with $\dim \Sigma_0 = 2n$ (see [10]). We remark that Σ_0 has the equations $\{y^{(1)i} = 0, \ \ldots, \ y^{(k-1)i} = 0 \ (i = 1, \ldots, n)$ and (x^i, p_i) are the local coordinates of the points $(x, p) \in \Sigma_0$.

Let us denote the restrictions of ω and H to the submanifold Σ_0 by ω_0 and H_0, respectively. In a point $u = (x, p) \in \Sigma_0$ the tangent space $T_u\Sigma_0$ has the natural basis $\left\{ \frac{\partial}{\partial x^i}\Big|_u, \frac{\partial}{\partial p_i}\Big|_u \right\}$ and natural cobasis $\left\{ dx^i\big|_u, dp_i\big|_u \right\}$. Then the pair (M, H_0) is a classical Hamilton space of first order having the fundamental tensor field $g^{ij}(x, 0, \ldots, 0, p) = \frac{1}{2} \frac{\partial^2 H}{\partial p_i \partial p_j}$,

(Σ_0, ω_0) is a symplectic manifold and $\{\ ,\ \}_0$, defined by

$$\{f, g\}_0 = \frac{\partial f}{\partial x^i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial x^i}, \tag{3.3}$$

is a Poisson structure on Σ_0.

Following the ideas from [9], [10], [14] we have:

Theorem 3.1. *i) There exists an unique vector field X_{H_0} on Σ_0 with the property*

$$i_{X_{H_0}}\omega_0 = -dH_0. \tag{3.4}$$

ii) X_{H_0} has the local form

$$X_{H_0} = \frac{\partial H_0}{\partial p_i}\frac{\partial}{\partial x^i} - \frac{\partial H_0}{\partial x^i}\frac{\partial}{\partial p_i}. \tag{3.5}$$

The integral curves of vector field X_{H_0} are given by the "Σ_0-canonical equations" of the space $H^{(k)n}$ (Hamilton-Jacobi equations):

$$\frac{dx^i}{dt} = \frac{\partial H_0}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H_0}{\partial x^i}, \quad y^{(1)i} = 0, \ \dots, \ y^{(k-1)i} = 0. \tag{3.6}$$

iii) The following formula holds:

$$\{f, g\}_0 = \omega_0(X_f, X_g), \quad \forall f, g \in C^\infty(\Sigma_0). \tag{3.7}$$

Corollary 3.1. *The dynamical system (Σ_0, X_{H_0}) is a Hamiltonian system and the symplectic form ω_0 is an invariant form for Hamiltonian system (Σ_0, X_{H_0}), with the Hamiltonian H_0.*

Corollary 3.2. *H_0 is constant along the integral curves of X_{H_0}, that is H is a conservation law for Hamiltonian system (Σ_0, X_{H_0}).*

For obtaining new conservation laws for the Hamiltonian system (Σ_0, X_{H_0}) one can apply the Proposition 1.1 and it results:

Proposition 3.1. *If $Y \in \mathcal{X}(\Sigma_0)$ is a symmetry for X_{H_0} and $Z \in \mathcal{X}(\Sigma_0)$ is a Y-pseudosymmetry for X_{H_0} then*

$$\Phi = \omega_0(Y, Z) \tag{3.8}$$

is a conservation law for (Σ_0, X_{H_0}).

Particularly, if Y and Z are symmetries for X_{H_0} then Φ from (3.8) is a conservation law for (Σ_0, X_{H_0}).

If $Y = Y^k\frac{\partial}{\partial x^k} + \widetilde{Y}_k\frac{\partial}{\partial p_k}$ and $Z = Z^k\frac{\partial}{\partial x^k} + \widetilde{Z}_k\frac{\partial}{\partial p_k}$ then (3.8) becomes

$$\Phi = \begin{pmatrix} Y^k & \widetilde{Y}_k \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} Z^k \\ \widetilde{Z}_k \end{pmatrix} = \widetilde{Y}_k Z^k - Y^k \widetilde{Z}_k. \tag{3.9}$$

Corollary 3.3. *If $Y \in \mathcal{X}(\Sigma_0)$ is a X_{H_0}-pseudosymmetry for X_{H_0} then*

$$\Phi = \omega_0(X_{H_0}, Y) = -L_Y H_0 \tag{3.10}$$

or

$$\Phi = \frac{\partial H_0}{\partial x^k}Y^k + \frac{\partial H_0}{\partial p_k}\widetilde{Y}_k \tag{3.11}$$

is a conservation law for (Σ_0, X_{H_0}).

The previous theory can be extended to the other Poisson structures $\{\ ,\ \}_\alpha$, $(\alpha = 1, \ldots, k-1)$.

So, for $\alpha \in \{1, \ldots, k-1\}$ and the point $x_0 \in M$, fixed, let's consider the subset of $T^{*k}M$,

$$\Sigma_\alpha = \left\{ u = (x, y^{(1)}, \ldots, y^{(k-1)}, p) \in T^{*k}M \;\middle|\; \begin{array}{l} x = x_0, \\ y^{(\beta)i} = 0, \beta = 1, \ldots, k-1, \beta \neq \alpha \end{array} \right\}.$$

Then Σ_α is an immersed submanifold of the manifold $T^{*k}M$, with $\dim \Sigma_\alpha = 2n$. The local coordinates of a point $u \in \Sigma_\alpha$ are $(y^{(\alpha)i}, p_i)$ and the tangent space $T_u\Sigma_0$ has the natural basis $\left\{ \frac{\partial}{\partial y^{(\alpha)i}}\Big|_u, \frac{\partial}{\partial p_i}\Big|_u \right\}$ and natural cobasis $\left\{ dy^{(\alpha)i}\big|_u, dp_i|_u \right\}$.

If we consider the following geometrical object fields on the manifold Σ_α:

$$\begin{aligned} \theta_\alpha &= p_i dy^{(\alpha)i}, \\ \omega_\alpha &= d\theta_\alpha = dp_i \wedge dy^{(\alpha)i}. \end{aligned} \tag{3.12}$$

then the pair $(\Sigma_\alpha, \omega_\alpha)$ is a symplectic manifold.

Let us denote the restrictions of ω and H to the submanifold Σ_α by ω_α and H_α, respectively. Then the pair (M, H_α) is a classical Hamilton space of first order having the fundamental tensor field $g^{ij}(x_0, 0, \ldots, 0, y^{(\alpha)}, 0, \ldots, 0, p) = \frac{1}{2}\frac{\partial^2 H}{\partial p_i \partial p_j}$.

Following the ideas from [9], [10], [12], [14], [16] we have:

Theorem 3.2. *i) There exists an unique vector field* X_{H_α} *on* Σ_α *with the property*

$$i_{X_{H_\alpha}}\omega_\alpha = -dH_\alpha. \tag{3.13}$$

ii) X_{H_α} *has the local form*

$$X_{H_\alpha} = \frac{\partial H_\alpha}{\partial p_i}\frac{\partial}{\partial y^{(\alpha)i}} - \frac{\partial H_\alpha}{\partial y^{(\alpha)i}}\frac{\partial}{\partial p_i}. \tag{3.14}$$

The integral curves of vector field X_{H_α} *are given by the "*Σ_α*-canonical equations" of the space* $H^{(k)n}$ *(Hamilton-Jacobi equations):*

$$x^i = x_0^i,\ y^{(\beta)i} = 0,\ \beta = 1, \ldots, k-1,\ \beta \neq \alpha,\ \frac{dy^{(\alpha)i}}{dt} = \frac{\partial H_\alpha}{\partial p_i},\ \frac{dp_i}{dt} = -\frac{\partial H_\alpha}{\partial y^{(\alpha)i}}. \tag{3.15}$$

Corollary 3.4. *The dynamical system* $(\Sigma_\alpha, X_{H_\alpha})$ *is a dynamical Hamiltonian system with the Hamiltonian* H_α *and the symplectic form* ω_α *is an invariant form for* $(\Sigma_\alpha, X_{H_\alpha})$.

Corollary 3.5. H_α *is constant along the integral curves of* $X_{H\alpha}$, *that is* H *is a conservation law for the dynamical Hamiltonian system* $(\Sigma_\alpha, X_{H_\alpha})$.

For obtaining new conservation laws for the Hamiltonian system $(\Sigma_\alpha, X_{H_\alpha})$ one can apply again the Proposition 1.1 and we obtain:

Proposition 3.2. *If* $Y \in \mathcal{X}(\Sigma_\alpha)$ *is a symmetry for* X_{H_α} *and* $Z \in \mathcal{X}(\Sigma_\alpha)$ *is a* Y*-pseudosymmetry for* X_{H_α} *then*

$$\Phi = \omega_\alpha(Y, Z) \tag{3.16}$$

is a conservation law for $(\Sigma_\alpha, X_{H_\alpha})$.

Particularly, if Y and Z are symmetries for X_{H_α} then $\Phi = \omega_\alpha(Y, Z)$ from (3.16) is a conservation law for $(\Sigma_\alpha, X_{H_\alpha})$.

If $Y = Y^k \frac{\partial}{\partial y^{(\alpha)k}} + \widetilde{Y}_k \frac{\partial}{\partial p_k}$ and $Z = Z^k \frac{\partial}{\partial y^{(\alpha)k}} + \widetilde{Z}_k \frac{\partial}{\partial p_k}$ then (3.16) becomes

$$\Phi = \begin{pmatrix} Y^k & \widetilde{Y}_k \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} Z^k \\ \widetilde{Z}_k \end{pmatrix} = \widetilde{Y}_k Z^k - Y^k \widetilde{Z}_k. \tag{3.17}$$

Corollary 3.6. *If $Y \in \mathcal{X}(\Sigma_\alpha)$ is a X_{H_α}-pseudosymmetry for X_{H_α} then*

$$\Phi = \omega_\alpha(X_{H_\alpha}, Y) = -\mathcal{L}_Y H_\alpha \tag{3.18}$$

or

$$\Phi = \frac{\partial H_\alpha}{\partial y^{(\alpha)k}} Y^k + \frac{\partial H_\alpha}{\partial p_k} \widetilde{Y}_k \tag{3.19}$$

is a conservation law for $(\Sigma_\alpha, X_{H_\alpha})$.

4 The case of Cartan spaces of order $k \geq 1$

As we know from the books of Acad. Radu Miron [9], [12] the "dual" of the Lagrange space of order $k \geq 1$, $L^{(k)n}$, via Legendre transformation, is a Hamilton space of order $k \geq 1$, $H^{(k)n}$. Thus, the "dual" of the Finsler space of order $k \geq 1$, $F^{(k)n}$, is a Cartan space of order $k \geq 1$, $\mathcal{C}^{(k)n}$ (see [8], [12]).

Definition 4.1. (R. Miron, [12]) *A Cartan space of order $k \geq 1$ is a pair $\mathcal{C}^{(k)n} = (M, K(x, y^{(1)}, \ldots, y^{(k-1)}, p))$ for which the following axioms hold:*

1^0 K is a real function on the manifold $T^{*k}M$, differentiable on $\widetilde{T^{*k}M}$ and continuous on zero section of the projection $\pi^{*k} : T^{*k}M \to M$.

2^0 $K > 0$ on $T^{*k}M$.

3^0 K is positively k-homogeneous on the fibres of the bundle $T^{*k}M$, i.e.

$$K(x, ay^{(1)}, \ldots, a^{k-1}y^{(k-1)}, a^k p) = a^k K(x, y^{(1)}, \ldots, y^{(k-1)}, p), \quad \forall a \in \mathbb{R}^+. \tag{4.1}$$

4^0 The Hessian of K^2, with respect to the momenta p_i, having the elements

$$g^{ij}(x, y^{(1)}, \ldots, y^{(k-1)}, p) = \frac{1}{2} \frac{\partial^2 K^2}{\partial p_i \partial p_j} \tag{4.2}$$

is positively defined.

From (4.2) we deduce that g^{ij} is contravariant tensor of order two, symmetric and nondegenerate, i.e. $rank||g^{ij}|| = n$ on $\widetilde{T^{*k}M}$.

The function K is called *fundamental* (or *metric*) function of $\mathcal{C}^{(k)n}$ and g^{ij} is *the fundamental tensor* of this space.

Obviously, if $\mathcal{C}^{(k)n} = (M, K)$ is a Cartan space of order k, then $H^{(k)n} = (M, K^2)$ is a Hamilton space of order k having the same fundamental tensor g^{ij} as the space $\mathcal{C}^{(k)n}$. $H^{(k)n}$ will be called the Hamilton space associated to the Cartan space of order k, $\mathcal{C}^{(k)n}$. All geometrical properties of $H^{(k)n} = (M, K^2)$ are geometrical properties of $\mathcal{C}^{(k)n} = (M, K)$.

So, using the canonical presymplectic structure ω from (2.2) and the submanifolds

Σ_0, Σ_{α}, $(\alpha = 1, \ldots, k-1)$ of $T^{*k}M$, from the previous section, we will obtain again all results from the previous section for the differentiable Hamiltonian $H = K^2$, where K is the fundamental function of a Cartan space of order k, $\mathcal{C}^{(k)n} = (M, K(x, y^{(1)}, \ldots, y^{(k-1)}, p))$.

5 The case of Generalized Hamilton and Cartan spaces of order $k \geq 1$

The results from previous sections can be written for the cases of Generalized Hamilton spaces and Generalized Cartan spaces of order $k \geq 1$, for which the Hamiltonian is represented by the *absolute energy* of the space, $\mathcal{E} = g^{ij}p_ip_j$.

Definition 5.1. (R. Miron, [12]) *A generalized Hamilton space of order k is a pair* $GH^{(k)n} = (M, g^{ij}(x, y^{(1)}, \ldots, y^{(k-1)}, p))$, *where*

1° g^{ij} is a d-tensor field of type (2,0), symmetric and nondegenerate on the manifold $\widetilde{T^{*k}M}$,

$$rank \left\| g^{ij} \right\| = n. \tag{5.1}$$

2° The quadratic form $g^{ij}X_iX_j$ has a constant signature on $\widetilde{T^{*k}M}$.

The tensor g^{ij} is called *fundamental* for the space $GH^{(k)n}$. The absolute energy $\mathcal{E}(x, y^{(1)}, \ldots, y^{(k-1)}, p)$ is a differentiable Hamiltonian uniquely determined by the fundamental tensor g^{ij} of the space $GH^{(k)n}$. It allows to determine the Hamilton-Jacobi equations of the space (for more details, see the book of Acad. Radu Miron [12]).

So, all results from section 3 are true for the differentiable Hamiltonian $\mathcal{E}$.

Definition 5.2. (R. Miron, [12]) *A generalized Cartan space of order k is a generalized Hamilton space of order k,* $GH^{(k)n} = (M, g^{ij})$, *for which the fundamental tensor g^{ij} satisfies the axioms:*

1° g^{ij} is positively defined on $\widetilde{T^{*k}M}$.

2° g^{ij} is 0-homogeneous on the fibres of the dual bundle $(T^{*k}M, \pi^{*k}, M)$.

We denote $GC^{(k)n} = (M, g^{ij})$ a generalized Cartan space of order k.

Taking into account that the absolute energy $\mathcal{E} = g^{ij}p_ip_j$ is $2k$-homogeneous on the fibres of the bundle $T^{*k}M$ (see [12]), we have that all results from Section 3 remain true for the differentiable Hamiltonian $\mathcal{E}$ associated to a generalized Cartan space of order k.

Acknowledgments. I wish to express my deep gratitude to Academician Radu Miron for the essential support granted me in the elaboration of this paper.

My sincere thanks go to Professor Petre Stavre for valuable discussions on the contents of this paper.

References

[1] Arnold, V. I., *Metodele matematice ale mecanicii clasice*, Ed. Stiinţifică şi Enciclopedică, Bucureşti, 1980

[2] Crampin, M., Pirani, F. A. E., *Applicable differential geometry*, London Math. Society, Lectures Notes Series, no. 59, Cambridge University Press, 1986

[3] Crâşmăreanu, M., *Conservation Laws Generated by Pseudosymmetries with Applications to Variational Dynamical Systems*, Preprint Seminarul de Mecanică, Universitatea de Vest Timişoara

[4] ten Eikelder, H. M. M., *Symmetries for Dynamical and Hamiltonian Systems*, CWI Tracts, no. 17, Amsterdam, 1985

[5] Jones, G. L., *Symmetry and Conservation Laws of Differential Equations*, Il Nuovo Cimento, 112 (1997), no. 7, 1053-1059

[6] Miron, R., Anastasiei, M., *The Geometry of Lagrange Spaces: Theory and Applications*, Fundamental Theories of Physics, no. 59, Kluwer Academic Publishers, 1994

[7] Miron, R., *The Geometry of Higher-Order Lagrange Spaces. Applications to Mechanics and Physics*, Fundamental Theories of Physics, no. 82, Kluwer Academic Publishers, 1997

[8] Miron, R., *The Geometry of Higher-Order Finsler Spaces*, Hadronic Press, Inc., USA, 1998

[9] Miron, R., Hrimiuc, D., Shimada, H., Sabău, V.S., *The Geometry of Lagrange and Hamilton Spaces*, Fundamental Theories of Physics, Kluwer Academic Publishers, 2001

[10] Miron, R., *On the Geometrical Theory of Higher-Order Hamilton Spaces*, Proceedings of the Colloquium on Differential Geometry, Debrecen, Hungary, 231-236, 2000

[11] Miron, R., *Hamilton Spaces of Order* $k \geq 1$, Int. Journal of Theoretical Physics, 2000 (to appear)

[12] Miron, R., *The Geometry of Higher-Order Hamilton Spaces. Applications to Hamiltonian Mechanics*, Fundamental Theories of Physics, Kluwer Academic Publishers, 2002 (to appear)

[13] Munteanu, F., *A Study on the Dynamical Systems in the Lagrangian and Hamiltonian Formalism*, Proceedings of SSMR Conference, Braşov, 2001 (to appear)

[14] Munteanu, F., *Dynamical Systems on Hamilton Spaces of First and Second Order*, Proceedings of the Fifth International Workshop on Differential Geometry and Its Applications, Timişoara, 2001 (to appear)

[15] Munteanu, F., *Conservation Laws for Dynamical Hamiltonian Systems on Hamilton Spaces of Order* $k \geq 1$, Proccedings of the Ninth Symposium of Mathematics and Its Applications, Timişoara, 2001 (to appear)

[16] Munteanu, F., *Spaţii Lagrange şi spaţii Hamilton. Aplicaţii la sisteme dinamice Hamiltoniene*, Ph. D. Thesis, Craiova, Romania, 2002 (in Romanian)

[17] Stavre, P., *Capitole speciale de geometrie diferenţială. Varietăţi pseudo-riemanniene*, Ed. Radical, Craiova, 2001