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Existence and Uniqueness of Solutions of Quasilinear Stochastic Delay Differential Equations in a Hilbert Space

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Abstract. In this paper we prove the theorem on existence and uniqueness of solutions of quasilinear stochastic delay differential equations in a Hilbert space using Schauder–Tychonov fixed point theorem. The result is illustrated with an example.

1 Introduction

Random differential and integral equations play an important role in characterizing of many social, physical, biological and engineering problems. For example, a stochastic model for drug distribution in a biological system was described by Tsokos and Padgett [22] to a closed system with a simplified heart, one organ or capillary bed, and recirculation of the blood with a constant rate of flow, where the heart is considered as a mixing chamber of constant volume. In 1975, Hida introduced the theory of white noise [11] so that, for each t, $w'(t)$ is a generalized function on the space $S'(R)$ of tempered distributions. It is a fact that $w'(t) = \partial_t + \partial_t^*$ when $w'(t)$ is regarded as a multiplication operator. Here ∂_t is the white noise differential operator and ∂_t^* is its adjoint. For details see [12], [15], [16]. White noise is usually regarded as the informal time derivative $w'(t)$ of a Brownian motion or Wiener process $w(t)$. In the Itô theory of stochastic integration an integral with respect to $w'(t)$ is rewritten as one with respect to $dw(t)$, that is,

$$\int_a^b \varphi(t)w'(t)dt = \int_a^b \varphi(t)dw(t).$$

The Itô integral $\int_a^b \varphi(t)dw(t)$ is defined for any stochastic process $\varphi(t)$ which satisfies the conditions: (1) φ is nonanticipating and (2) almost all sample paths of φ belong to $L^2[a,b]$. Moreover, $\int_a^b \varphi(t)dw(t) \in L^2(\Omega)$ if and only if $\varphi \in L^2([a,b]\times\Omega)$. In fact, the following equality holds

$$E\left|\int_a^b \varphi(t)dw(t)\right|^2 = E\int_a^b |\varphi(t)|^2 dt.$$

In an infinite dimensional Banach space, the semigroup theory gives a unified treatment of a wide class of stochastic parabolic, hyperbolic and functional differential equations. The Banach contraction principle is widely used to study the existence of solutions of stochastic evolution equations in Banach spaces (see Ahmed [1]) and in Hilbert spaces (see Ahmed and Ding [2], Loon and Nualart [17]). Liu [18] obtained Carathéodory approximate solutions for a class of semilinear stochastic evolution equation in Hilbert space.

The most important problems examined up to now is the one concerning the existence of solutions of considered equations. The basic tools used in solving this problem were mostly the method of successive approximations and the Banach fixed point principle (see for example [4], [13], [19], [20], [21]). Recently, global existence of solutions for a semilinear stochastic delay evolution equations with nonlocal conditions have been studied by Balasubramaniam and Ntouyas [3] using a Leray–Schauder Alternative analysis approach.

At the present time, it is well known that the random or stochastic differential equation is a very important one for the formulation and analysis in mechanical, electrical and control engineering and physical and biological sciences. Furthermore, the fundamental importance is pointed out also in the investigations to economic and social sciences. Theoretical treatments of such problems can be found in earlier contributions by many authors (for example see [7], [8], [9], [18]).

The aim of this paper is to prove an existence and uniqueness of solutions for quasilinear stochastic delay differential equations in a Hilbert space. The approach is based on the new fixed point theorem called Schauder - Tychonov theorem.

2 Preliminaries

We denote $L(Y, Y')$, the Hilbert space of all bounded linear operators from Y to Y'. The symbol $\|\cdot\|$ denotes the norm of all the spaces of bounded linear operators. It also denotes the sup–norm of any bounded continuous function.

Unless otherwise specified, $L(Y, Y')$ will be assumed to be associated with the uniform operator topology. Let $M \subset Y$ and $A : M \to Y'$ be given. Then A is said to be compact if A is continuous on M and maps bounded subsets of M onto relatively compact subsets of Y'. Let $J \subset \mathbb{R}$ $(= (-\infty, \infty))$ be a bounded interval and let the operator $A : J \times Y^2 \to Y'$ be given. We say that $A(t, u, v)$ is continuous in tY^2-uniformly in (u, v), if for every bounded subset M of Y we have

$$\lim_{\substack{t \to t_0 \\ t \in J}} \sup_{u,v \in M} \|A(t, u, v) - A(t_0, u, v)\|^2 = 0, \tag{1}$$

for every $t_0 \in J$. We denote by $\mathbb{C}(J, Y)$ the space of all continuous functions $z : J \to Y$ with the sup-norm $\|z(t)\|^2 = \sup\{E|z(t)|^2;\ t \in J\}$. If $J = [-r, 0]$, then this space is denoted by $\mathbb{C}$.

In this paper we establish a local existence result for delay problem, for each fixed random value $z \in \mathbb{C}([0, T), Y)$

$$\begin{aligned} dx(t) &+ A(t, z(t), z(t-r))x(t)dt \\ &= f(t, z(t), z(t-r))dt + G(t, z(t), z(t-r))dw(t),\ t \in [0, T) \end{aligned} \tag{2}$$

$$x(t) = \phi(t),\ t \in [-r, 0], \tag{3}$$

where the operator $A(t, u, v)w$ is linear and bounded in w and continuous on $[0, T) \times Y^2$.

In order to prove the existence result assume the following conditions:

(S_1) $A(t, u, v) \in L(Y, Y)$ for every $(t, u, v) \in [0, T) \times Y^2$. Moreover, $A(t, u, v)$ is compact in (u, v) and continuous in tY^2-uniformly in (u, v).

(S_2) $f : [0, T) \times Y^2 \to Y$ and $f(t, u, v)$ is compact in (u, v) and continuous in tY^2-uniformly in (u, v).

(S_3) $G : [0, T) \times Y^2 \to Y$ and $G(t, u, v)$ is compact in (u, v) and continuous in tY^2-uniformly in (u, v).

(S_4) $\phi : [-r, 0] \to Y$ is the initial datum.

3 Main Result

Theorem 3.1. *Let the assumptions (S_1)–(S_4) be satisfied. Then there exists a number $T_1 \in (0, T]$ and a continuous function $x : [-r, T_1] \to Y$ such that $x(t) = \phi(t)$, $t \in [-r, 0]$ and $x(t)$ is strongly differentiable and satisfies the differential equation* (2)*–*(3) *on $J = [0, T_1]$.*

Proof. Given $z \in \mathbb{C}([-r, T_1], Y)$ for some $T_1 \in [0, T)$, we let $X_z(t)$, $t \in J$, $X_z(0) = I$, denote the fundamental operator of the equation (see [14])

$$dx(t) + A(t, z(t), z(t-r))x\,dt = 0,\ x(0) = \phi(0). \tag{4}$$

Then $X_z \in \mathbb{C}(J, L(Y, Y))$ and X_z is the unique continuously differentiable solution of the problem

$$dX(t) + A(t, z(t), z(t-r))X\,dt = 0,\ X(0) = I,\ t \in J. \tag{5}$$

Moreover, $x_z^{-1} \in \mathbb{C}((J, L(Y, Y))$ and X_z^{-1} is the unique continuously differentiable solution of the problem

$$dX(t) - AS(t, z(t), z(t-r))X\,dt = 0,\ X(0) = I,\ t \in J. \tag{6}$$

The problem (2)–(3) has a unique solution $x_z(t)$, $t \in [-r, T_1)$ such that

$$\begin{aligned} x_z(t) &= X_z(t)\phi(0) + \int_0^t X_z(t)X_z^{-1}(s)f(s, z(s), z(s-r))ds \\ &+ \int_0^t X_z(t)X_z^{-1}(s)G(s, z(s), z(s-r))dw(s),\ \text{for every } t \in J. \end{aligned} \tag{7}$$

We assume the following additional hypothesis:

(H_1) There is a positive constant k_1 such that the fundamental operator solution X_z satisfies

$$\|X_z(t)\|^2 \le k_1 \ \text{ and } \ \|X_z^{-1}(t)\|^2 \le k_1.$$

(H_2) The operators $A(t,u,v)$, $f(t,u,v)$ and $G(t,u,v)$ are compact, continuous tY^2-uniformly in (u,v) and satisfy equation (1) with

$$\|A(t,z(t),z(t-r))\|^2 \le k_2,\ \|f(t,z(t),z(t-r))\|^2 \le k_3 \text{ and}$$
$$\|G(t,z(t),z(t-r))\|^2 \le k_4,$$

where k_2, k_3 and k_4 are positive constants.

Let

$$M = \Big\{z \in \mathbb{C}([-r,T_1],Y) : z(t) = \phi(t),\ t\in[-r,0],\ \|z\|^2 \le L \text{ and}$$
$$\|z(t)-z(t')\|^2 \le N(t)|t-t'|,\ t,t' \in J\Big\},$$

where,

$$N(t) = 9k_1k_2|t-t'|\|\phi(0)\|^2 + 9k_1^2\{k_2|t-t'|T_1(k_3T_1+k_4)+k_3|t-t'|+k_4\},$$
$$L = 9k_1\|\phi(0)\|^2 + 9k_1^2T_1(k_3T_1+k_4).$$

The set M is non empty, because the function $z : [-r,T_1] \to Y$ with $z(t) = \phi(t)$, $t \in [-r,0]$ and $z(t) = \phi(0)$, $t \in J$, belongs to M.

Let $\Phi : M \to \mathbb{C}([-r,T_1],Y)$ be the operator that maps $z \in M$ into x_z by $\Phi x_z(t) = \phi(t)$, for $t \in [-r,0]$

$$\Phi x_z(t) = X_z(t)\phi(0) + \int_0^t X_z(t)X_z^{-1}(s)f(s,z(s),z(s-r))ds$$
$$+ \int_0^t X_z(t)X_z^{-1}(s)G(s,z(s),z(s-r))dw(s),\ \text{for } t\in J.$$

In order to apply the Schauder–Tychonov Theorem on M, we first show that $\Phi M \subset M$. In fact, given $z \in M$, we have

$$\|x_z(t)\|^2 \le \phi(t),\ t\in[-r,0]$$

and for $t \in J$

$$\begin{aligned}\|x_z(t)\|^2 &\le 9\|X_z(t)\|^2\|\phi(0)\|^2 + 9\left\|\int_0^t X_z(t)X_z^{-1}(s)f(s,z(s),z(s-r))ds\right\|^2\\ &\quad + 9\left\|\int_0^t X_z(t)X_z^{-1}(s)G(s,z(s),z(s-r))dw(s)\right\|^2\\ &\le 9k_1\|\phi(0)\|^2 + 9k_1^2k_3T_1^2 + 9k_1^2k_4T_1,\\ \|x_z(t)\|^2 &\le L.\end{aligned}$$

Since $X_z(t)$ satisfies the equation (5) we have

$$\begin{aligned}\|X_z(t)-X_z(t')\|^2 &\le |t-t'|\left|\int_{t'}^t \|A(s,z(s),z(s-r))\|^2\|X_z(s)\|^2ds\right|\\ &\le k_1k_2|t-t'|.\end{aligned}$$

Using this and given $t,t' \in J$, we have

$$
\begin{aligned}
&\|x_z(t) - x_z(t')\|^2 \\
= &\ \Big\| [X_z(t) - X_z(t')]\phi(0) + [X_z(t) - X_z(t')] \int_0^t X_z^{-1}(s) f(s, z(s), z(s-r)) ds \\
&+ \int_{t'}^t X_z(t') X_z^{-1}(s) f(s, z(s), z(s-r)) ds \\
&+ [X_z(t) - X_z(t')] \int_0^t X_z^{-1}(s) G(s, z(s), z(s-r)) dw(s) \\
&+ \int_{t'}^t X_z(t') X z^{-1}(s) G(s, z(s), z(s-r)) dw(s) \Big\|^2 \\
\leq &\ 9k_1k_2|t-t'|^2\|\phi(0)\|^2 + 9k_1k_2|t-t'|^2(t-0)\int_0^t \|X_z^{-1}(s)\|^2\|f(s, z(s), z(s-r))\|^2 ds \\
&+ 9|t-t'| \left| \int_{t'}^t \|X_z(t')\|^2 \|X_z^{-1}(s)\|^2 \|f(s, z(s), z(s-r))\|^2 ds \right| \\
&+ 9k_1k_2|t-t'|^2 \int_0^t \|X_z^{-1}(s)\|^2 \|G(s, z(s), z(s-r))\|^2 ds \\
&+ 9 \left| \int_{t'}^t \|X_z(t')\|^2 \|X_z^{-1}(s)\|^2 \|G(s, z(s), z(s-r))\|^2 ds \right| \\
\leq &\ 9k_1k_2|t-t'|^2\|\phi(0)\|^2 + 9k_1^2k_2k_3|t-t'|^2T_1^2 \\
&+ 9k_1^2k_3|t-t'|^2 + 9k_1^2k_2k_4|t-t'|^2T_1 + 9k_1^2k_4|t-t'| \\
\leq &\ 9k_1k_2|t-t'|^2\|\phi(0)\|^2 + 9k_1^2k_2|t-t'|^2T_1(k_3T_1 + k_4) + 9k_1^2|t-t'|(k_3|t-t'| + k_4) \\
\leq &\ N(t)|t-t'|.
\end{aligned}
$$

Hence $\|x_z(t) - x_z(t')\|^2 \leq N(t)|t-t'|$. It follows that $\Phi M \subset M$.

To show that Φ is continuous, let $z_n, z \in M$ be given with $\|z_n - z\|^2 \to 0$, as $n \to \infty$. Then from hypothesis (H_2) with

$$
\begin{aligned}
&\|X_{z_n}(t) - X_z(t)\|^2 \\
\leq &\ t\int_0^t \|A(s, z_n(s), z_n(s-r))X_{z_n}(s) - A(s, z(s), z(s-r))X_z(s)\|^2 ds \\
\leq &\ 2t\int_0^t \|A(s, z_n(s), z_n(s-r)) - A(s, z(s), z(s-r))\|^2 \|X_{z_n}(s)\|^2 ds \\
&+ 2t\int_0^t \|A(s, z(s), z(s-r))\|^2 \|X_{z_n}(s) - X_z(s)\|^2 ds
\end{aligned}
$$

and Gronwall's inequality

$$
\|X_{z_n}(t) - X_z(t)\|^2 \leq 2T_1k_1 \int_0^{T_1} \|A(s, z_n(s), z_n(s-r)) - A(s, z(s), z(s-r))\|^2 ds \exp(2T_1^2k_2),
$$

for every $t \in J$. This shows that $\|X_{z_n} - X_z\|^2 \to 0$, as $n \to \infty$. Similarly, using (6), we can prove that $\|X_{z_n}^{-1} - X_z^{-1}\|^2 \to 0$, as $n \to \infty$. From the continuity of f and G wse see that $f(t, z_n(t), z_n(t-r))$ and $G(t, z_n(t), z_n(t-r))$ converges uniformly to $f(t, z(t), z(t-r))$ and $G(t, z(t), z(t-r))$ respectively on J. Using these facts, we can now obtain from (7) and the corresponding equation with z repalced by z_n, that $\|x_{z_n} - x_z\|^2 \to 0$ as $n \to \infty$. Consequently, Φ is continuous on M.

Before we show that ΦM is a relatively compact set, we first prove that the operators

$$A_1 : M \to \mathbb{C}(J, L(Y, Y)), f_1 : M \to \mathbb{C}(J, L(Y, Y)) \text{ and } G_1 : M \to \mathbb{C}(J, L(Y, Y))$$

with

$$\begin{aligned} (A_1 z)(t) &= A(t, z(t), z(t-r)), \\ (f_1 z)(t) &= f(t, z(t), z(t-r)), \\ (G_1 z)(t) &= G(t, z(t), z(t-r)) \end{aligned}$$

are compact. For this, let $\{z_n\}$ be a sequence in M. We first observe that $\|A(t, z_n(t), z_n(t-r))\|^2 \leq k_2$, $n = 1, 2, ...$, $t \in J$. Given $t, t_0 \in J$, we find

$$\begin{aligned} &\|A\,(t, z_n(t), z_n(t-r)) - A(t_0, z_n(t_0), z_n(t_0 - r))\|^2 \\ &\leq 2\|A(t, z_n(t), z_n(t-r)) - A(t, z_n(t), z_n(t-r))\|^2 \\ &\quad + 2\|A(t_0, z_n(t), z_n(t-r)) - A(t, z_n(t_0), z_n(t_0 - r))\|^2 \\ &\leq 2 \sup_{\substack{u \in Y, \|u\|^2 \leq L \\ v \in Y, \|v\|^2 \leq L}} \|A(t, u, v) - A(t_0, .u, v)\|^2 \\ &\quad + 2\|A(t_0, z_n(t), z_n(t-r)) - A(t_0, z_0(t_0), z_n(t_0 - r))\|^2, \end{aligned}$$

which, by the tY^2-uniform continuity of $A(t, u, v)$ and the uniform Lipschitz continuity of the functions z_n on $[-r, T_1]$, implies the equicontinuity of the set of all functions $A_n(t) \equiv A(t, z_n(t), z_n(t-r))$, $t \in J$, $n = 1, 2, ...$ Now, let $t_0 \in J$ be given. Then since $\{z_n(t_0)\}$ is a bounded sequence, the compactness of $A(t_0, u, v)$ in (u, v) implies the relative compactness of the set $\{A(t_0, z_n(t_0), z_n(t_0 - r))\}$. Consequently, the operator A_1 is compact. A similar argument proves the compactness of f_1 and G_1. Thus, given a sequence $\{z_n\} \subset M$, there exists a subsequence $\{z_{n_k}\}$ of $\{z_n\}$ such that

$$\begin{aligned} &A(t, z_{n_k}(t), z_{n_k}(t-r)) \to A(t), \\ &f(t, z_{n_k}(t), z_{n_k}(t-r)) \to f(t), \\ &G(t, z_{n_k}(t), z_{n_k}(t-r)) \to G(t), \end{aligned}$$

uniformly on J, as $k \to \infty$. Let $X(t)$ denote the fundamental operator for the problem

$$dx(t) + A(t)x(t)dt = 0, \; x(0) = \phi(0).$$

Then,

$$x(t) = X(t)\phi(0) + \int_0^t X(t)X^{-1}(s)f(s)ds + \int_0^t X(t)X^{-1}(s)G(s)dw(s), \; t \in J,$$

is the unique solution of the problem

$$dx(t) + A(t)x(t)dt = f(t)dt + G(t)dw(t), \; x(0) = \phi(0), \; t \in J.$$

It is easy to see now that $X_{z_{n_k}}(t) \to X(t)$ and $X^{-1}_{z_{n_k}} \to X^{-1}(t)$ uniformly on J. It follows that $Z_{z_{n_k}} \to X(t)$ uniformly on J. Since $Z_{z_{n_k}}(t) = \phi(t)$, $t \in [-r, 0]$, we have actually shown the compactness of ΦM. Any fixed point of the operator Φ in M is a solution to our problem.

4 Example

Using the above theorem, we can study the existence of solutions of the systems of the form

$$\begin{aligned} &d(E(t)z(t)) + A(t, z(t), z(t-r))z(t)dt \\ &\quad = f(t, z(t), z(t-r))dt + G(t, z(t), z(t-r))dw(t),\ t \in [0, T) \end{aligned} \tag{8}$$

$$E(t)z(t) = \phi(t) \text{ on } [-r, 0] \tag{9}$$

when the following additional assumptions hold:

(i) For each $t \in]-r, T]$, $E(t)$ is linear, closed and densely defined with domain $D(E)$ (independent of t) in $D(A)$ and range Y. Moreover, for each $t \in [-r, T)$, $E^{-1}(t) : Y \to X$ exists and is compact while $E^{-1}(t)z$ is continuous in t for each $z \in Y$.

(ii) For each $(t, z, v) \in [0, T) \times Y^2$,

$$A(t, E^{-1}(t), E^{-1}(t-r)v)E^{-1}(t) \in L(Y, Y),$$

is continuous in (t, z, v) with its continuity tY^2-uniform in (z, v).

(iii) For each $(t, z, v) \in [0, T) \times Y^2$,

$$f(t, E^{-1}(t)z, E^{-1}(t-r)v) \in Y$$

is continuous in (t, z, v) with its continuity tY^2-uniform in (z, v).

(iv) For each $(t, z, v) \in [0, T) \in Y^2$,

$$G(t, E^{-1}(t)z, E^{-1}(t-r)v) \in Y$$

is continuous in (t, z, v) with its continuity tY^2-uniform in (z, v), $\phi : [-r, 0] \to Y$ is the initial datum.

For this, consider the problem

$$\begin{aligned} &dv(t) + A(t, E^{-1}(t)v(t), E^{-1}(t-r)v(t-r))E^{-1}(t)v(t)dt \\ &\quad = f(t, E^{-1}(t)v(t), E^{-1}(t-r)v(t-r))dt \\ &\qquad + G(t, E^{-1}(t)v(t), E^{-1}(t-r)v(t-r))dw(t),\ t \in [0, T), \end{aligned} \tag{10}$$

$$v(t) = \phi(t) \text{ on } [-r, 0]. \tag{11}$$

If $v(t)$, $t \in [-r, T_1]$, for some $T_1 \in [0, T)$ is a solution of (10)–(11), then $z(t) = E^{-1}(t)v(t)$, $t \in [-r, T_1]$ satisfies the equation (8)-(9). Therefore the existence of solutions of (8)-(9) is equivalent to that of (10)-(11), and the proof is similar. Hence it is omitted.

Example 4.1. Let $\Omega \in \mathbb{R}^n$, $n \geq 2$, be a bounded domain with smooth boundary $\partial\Omega$ and closure $\overline{\Omega}$. Given an integer $m \geq 0$ and a real number $p \in (0, \infty)$ we denote by $L^p = (L^p(\Omega), \|\cdot\|_p)$, $W^{m,p} = (W^{m,p}(\Omega), \|\cdot\|_{m,p})$ the usual Sobolev spaces. We consider the following two elliptic operators

$$a(u, t, z)v = \sum_{|\alpha| \leq 2h} b_\alpha(u, t, \mu(z))D^\alpha v$$

$$e(u)z = \sum_{|\alpha| \leq 2m} c_\alpha(u)D^\alpha z,$$

where $\mu(z) = \{D^\alpha z : |\alpha| \leq q\}$ with q is defined below and m, h are two positive integers with $h \leq m$. We assume that $(2m-1)p > n$ and $q = (2m-1) - \left(\frac{n}{p}\right)$, and take d_0 and d_1 as in [5].

The functions $b_\alpha : \overline{\Omega} \times \mathbb{R}_+ \times \mathbb{R}^{d_0} \to \mathbb{R}$, $|\alpha| \leq 2h$, $c_\alpha \leq 2m$, are continuous, uniformly bounded and such that for $(u, t, \mu) \in \Omega \times \mathbb{R}_+ \times \mathbb{R}^{d_0}$, $0 \neq (\beta_1, \beta_2, ..., \beta_n) \in \mathbb{R}^n$,

$$\left.\begin{array}{l} \sum_{|\alpha|=2h} b_\alpha(u,t,\mu)\beta_1^{\alpha}...\beta_n^{\alpha_n} \neq 0 \\ \sum_{|\alpha|=2m} c_\alpha(u)\beta_1^{\alpha_1}...\beta_n^{\alpha_n} \neq 0 \end{array}\right\}.$$

Moreover, there exists constants $k_5, k_6 \geq 0$ such that

$$|b_\alpha(u,t,\mu) - b_\alpha(u,t',\mu')| \leq k_5|t-t'| + k_6|\mu - \mu'|$$

for every $u \in \Omega$, $t, t' \in \mathbb{R}_+$, $\mu, \mu' \in \mathbb{R}^{d_0}$, where $|\mu| = \sum_{|\alpha|\leq q} |\mu_\alpha|$. The boundary operators $\{E_i\}_{i=1}^m$, $\{E_i^1\}_{i=1}^k$ and the spaces $W^{2m,p}(\Omega; \{E_i\}_{i=1}^m)$, $W^{2h,p}(\Omega; \{E_i^1\}_{i=1}^k)$ are defined as in [5], [10]. We are going to study the problem

$$\begin{aligned} &\frac{\partial}{\partial t}(e(u)z(u,t)) + a(u,t,z(u,t-r))z(u,t) \\ &\quad = b(u,(D^a z(u,t-r))_{|\alpha|\leq 2m-1}, z(u,t)) \\ &\qquad + g(u,(D^a z(u,t-r))_{|\alpha|\leq 2m-1}, z(u,t)),\ (u,t) \in \Omega \times (0,\infty), \end{aligned} \tag{12}$$

with

$$\begin{array}{rcll} E_i(u,t) &=& 0, & (u,t) \in \partial\Omega \times (0,\infty),\ 1 \leq i \leq m, \\ e(u)z(u,t) &=& \phi(u,t), & (u,t) \in \Omega \times [-r,0]. \end{array}$$

For the operators b and g we assume the following:

$b, g : \Omega \times \mathbb{R}^{d_1} \times \mathbb{R} \to \mathbb{R}$ are continuous, and there exist $b_1, g_1 \in L^p$ and constants $k_7, k_8 > 0$ such that for $(u, \mu, z) \in \Omega \times \mathbb{R}^{d_1} \times \mathbb{R}$,

$$|b(u,\mu,z)| \leq b_1(u) + k_7 \sum_{|\alpha|\leq 2m-1} |\mu_\alpha|,$$

$$|g(u,\mu,z)| \leq g_1(u) + k_8 \sum_{|\alpha|\leq 2m-1} |\mu_\alpha|.$$

For the function ϕ in (12) we assume that $\phi(\cdot, t) \in L^p$, $t \in [-r, 0]$, and is Lipschitz continuous in t uniformly wit respect to $u \in \Omega$.

We set $Y = W^{2m-1,p}$, $X = L^p$ for $(u,t) \in \Omega \times [0,T]$ and define

$$(Ez)(u) = \sum_{|\alpha|\leq 2m} c_a(u)(D^a z)(u), \quad z \in D(E) = W^{2m,p}(\Omega; \{E_i\}_{i=1}^m),\ u \in \Omega,$$

$$(A(t,z)v)(u) = \sum_{|\alpha|\leq 2h} b_a(u,t,((D^a z)(u))_{|\alpha|\leq q})(D^a v)(u), \quad z \in W^{2m-1,p},$$

$$v \in D(A) = W^{2h,p}(\Omega; \{E_i\}_{i=1}^k) \cap Y \supset D(E),\ (u,t) \in \Omega \times [0,\infty),$$

$$f(z,v)(u) = b(u,((D^a z)(u))_{|\alpha|\leq q}, v(u)), \quad z, v \in W^{2m-1,p},$$

$$(G(z,v))(u) = g(u, ((D^a z)(u))_{|\alpha| \leq 2m-1}, v(u)), \quad z, v \in W^{2m-1,p}, \quad u \in \Omega.$$

In addition, if we assume that the operator E is bijective, then the equation (8)-(9), corresponding to problem (12), will have at least one solution $z(t)$ on $[-r, \infty)$. The condition (ii) holds and all other properties of E, A, f, G hold true in the present setting (see [5], [10]). Hence, by Theorem 3.1, (8)-(9) has unique solution.

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