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Controllability of Delay Integrodifferential Systems in Banach Spaces

K.Balachandran and R.Sakthivel

Abstract: Sufficient conditions for controllability of delay integrodifferential systems in a Banach space are established. The results are obtained by using the Schaefer fixed-point theorem.

Key Words: Controllability, integrodifferential system, fixed point theorem.

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1.Introduction:

Controllability of linear and nonlinear systems represented by ordinary differential equations in finite-dimensional space has been extensively studied. Several authors have extended the concept to infinite-dimensional systems in Banach spaces with bounded operators. Naito [9,10] has studied the controllability of semilinear systems whereas Yamamoto and Park [14]considered the same problem for parabolic equation with uniformly bounded nonlinear term. Chukwu and Lenhart [3] have studied the controllability of nonlinear systems in abstract spaces. Do [4] and Zhou [15] discussed the approximate controllability for a class of semilinear abstract equations. Naito [11] established the controllability for nonlinear Volterra integrodifferential systems. Nakagiri and Yamamoto [6] studied the controllability for delay systems and Kwun et al [5] studied the approximate controllability for delay Volterra systems with bounded linear operators. Recently Balachandran et al [1,2] established sufficient conditions for the controllability of nonlinear integrodifferential systems in Banach spaces by using Schauder's fixed-point theorem. The purpose of this paper is to study the controllability of delay integrodifferential systems in Banach spaces by suitably applying the Schaefer fixed-point theorem.

2. Preliminaries:

Consider the delay integrodifferential system

$$\begin{aligned} x'(t) &= Ax(t) + (Bu)(t) \\ &\quad + f(t, x(\sigma_1(t)), \int_0^t l(t, s)g(s, x(\sigma_2(s)))ds), \ t \in J = [0, b], \qquad (1) \\ x(t) &= \phi(t), \qquad t \in [-h, 0] \end{aligned}$$

where $\sigma_i(t) = t - h_i(t)$ with $h_i(t) \geq 0$, h=max$\{h_i(t) : t \in J\}$, $h_i : J \to J$, i=1,2 are continuous functions , the state x(.) takes values in the Banach space X and the control function u(.) is given in $L^2(J,U)$, a Banach space of admissible control functions with U as a Banach space. A is the infinitesimal generator of a strongly continuous semigroup T(t), $t \geq 0$ in a Banach space X , $g : J \times X \to X$, $f : J \times X \times X \to X$, $l : J \times J \to R$ are given functions, $\phi \in Z = C([-h,0],X)$ and B is a bounded linear operator from U into X.

We need the following fixed point theorem due to Schaefer[13]

Schaefer Theorem: Let S be a convex subset of a normed linear space E and 0 ∈ S. Let F: S → S be a completely continuous operator and let

$$\zeta(F) = \{x \in S; x = \lambda Fx \;\; for\ some \;\; 0 < \lambda < 1\}$$

Then either ζ(F) is unbounded or F has a fixed point.

The system (1) has a mild solution of the following form [12]

$$\begin{aligned} x(t) &= T(t)\phi(0) + \int_0^t T(t-s)[(Bu)(s) \\ &\quad + f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)]ds, \quad t \in J, \qquad (2) \\ x(t) &= \phi(t), \quad t \in [-h,0] \end{aligned}$$

In order to study the controllability problem of (1) we consider the following system as in [7,8]

$$\begin{aligned} x'(t) &= \lambda Ax(t) + \lambda(Bu)(t) \\ &\quad + \lambda f(t, x(\sigma_1(t)), \int_0^t l(t,s)g(s, x(\sigma_2(s)))ds), \;\; \lambda \in (0,1), \; t \in J. \qquad (3) \\ x(t) &= \phi(t), \qquad t \in [-h,0] \end{aligned}$$

Then for the system (3) there exists a mild solution of the following form

$$\begin{aligned} x(t) &= \lambda T(t)\phi(0) + \lambda \int_0^t T(t-s)[(Bu)(s) \\ &\quad + f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)]ds, \quad t \in J. \\ x(t) &= \phi(t), \quad t \in [-h,0] \end{aligned}$$

Definition:

The system (1) is said to be controllable on the interval J if for every continuous initial function $\varphi \in Z$, there exists a control $u \in L^2(J,U)$ such that the solution x(t) of (1) satisfies x(b) = x_1.

We assume the following hypothesis:

(i) T(t), t>0 is compact.

(ii) The linear operator W from $L^2(J,U)$ into X, defined by

$$Wu = \int_0^b T(b-s)Bu(s)ds$$

has an invertible operator W^{-1} defined on $L^2(J,U)/kerW$ and there exist positive constants M_2, M_3 such that $\|B\| \leq M_2$ and $\|W^{-1}\| \leq M_3$.

(iii) For each t ∈ J the function g(t,.) : X →X is continuous and for each $x \in X$ the function $g(.,x) : J \to X$ is strongly measurable.

(iv) For each $t \in J$ the function $f(t,.,.) : X \times X \to X$ is continuous and for each x,y ∈ X the function $f(.,x,y) : J \to X$ is strongly measurable.

(v) For every positive integer k there exists $\alpha_k \in L^1(0,b)$ such that for a.e t ∈J

$$\sup_{\|x\|,\|y\| \leq k} \|f(t,x,y)\| \leq \alpha_k(t)$$

(vi) There exists a continuous function $m : J \to [0,\infty)$ such that

$$\|g(t,x)\| \leq m(t)\Omega(\|x\|), \quad t \in J, x \in X$$

where $\Omega : [0,\infty) \to (0,\infty)$ is a continuous nondecreasing function.

(vii) There exists a continuous function $p : J \to [0,\infty)$ such that

$$\|f(t,x,y)\| \leq p(t)\Omega_0(\|x\| + \|y\|), \quad t \in J \quad x, y \in X,$$

where $\Omega_0 : [0,\infty) \to (0,\infty)$ is a continuous nondecreasing function.

(viii) There exists a constant L such that

$$\|l(t,s)\| \leq L \quad for \quad t \geq s \geq 0.$$

(ix)

$$\int_0^b \hat{m}(s)ds \leq \int_c^\infty \frac{ds}{\Omega_0(s) + \Omega(s)}$$

where $c = M_1(\|\phi(0)\|) + M_1 Nb, \qquad \hat{m}(t) = \max\{M_1 p(t), Lm(t)\},$
$M_1 = \sup\{\|T(t)\| : t \in J\}$ and

$$N = M_2 M_3[\|x_1\| + M_1\|\phi(0)\| + M_1 \int_0^b p(s)\Omega_0(\|x\| + L\int_0^s m(\tau)\Omega(\|x\|)d\tau)ds$$

3. Main Result

Theorem: If the hypothesis (i) - (ix) are satisfied, then the system (1) is controllable on J.
Proof:

Using the hypothesis (ii) for an arbitrary function x(.) define the control

$$u(t) = W^{-1}[x_1 - T(b)\phi(0) - \int_0^b T(b-s)f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)ds](t)$$

We shall now show that when using this control the operator defined by

$$\begin{aligned}(Fx)(t) &= T(t)\phi(0) + \int_0^t T(t-s)[(Bu)(s) \\ &\quad + f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)]ds, \quad t \in J\end{aligned}$$

has a fixed point. This fixed point is then a solution of equation (2).

Clearly (Fx) (b) = x_1 , which means that the control u steers the delay integrodifferential system from the initial function ϕ to x_1 in time T, provided we can obtain a fixed point of the nonlinear operator F.

First we obtain a priori bounds for the following equation

$$\begin{aligned}x(t) &= \lambda T(t)\phi(0) + \lambda \int_0^t T(t-\eta)BW^{-1}[x_1 - T(b)\phi(0) \\ &\quad - \int_0^b T(b-s)f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)ds](\eta)d\eta \\ &\quad + \lambda \int_0^t T(t-s)f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)ds\end{aligned}$$

we have

$$\begin{aligned}\|x(t)\| &\le M_1\|\phi(0)\| + \int_0^t \|T(t-\eta)\|M_2M_3[\|x_1\| + M_1\|\phi(0)\| \\ &\quad + M_1\int_0^b p(s)\Omega_0(\|x\| + L\int_0^s m(\tau)\Omega(\|x\|)d\tau)ds]d\eta \\ &\quad + M_1\int_0^t p(s)\Omega_0(\|x\| + L\int_0^s m(\tau)\Omega(\|x\|)d\tau)ds \\ &\le M_1\|\phi(0)\| + \int_0^t M_1Nds \\ &\quad + M_1\int_0^t p(s)\Omega_0(\|x\| + L\int_0^s m(\tau)\Omega(\|x\|)d\tau)ds \\ &\le M_1\|\phi(0)\| + M_1Nb + M_1\int_0^t p(s)\Omega_0(\|x\|\end{aligned}$$

$$+ L \int_0^s m(\tau)\Omega(\|x\|)d\tau)ds$$

Denoting by v(t) the right-hand side of the above inequality we have
$v(0) = M_1(\|\phi(0)\|) + M_1 Nb, \|x(t)\| \leq v(t)$ and

$$\begin{aligned} v'(t) &= M_1 p(t)\Omega_0(\|x\| + L \int_0^t m(\tau)\Omega(\|x\|)d\tau) \\ &\leq M_1 p(t)\Omega_0(v(t) + L \int_0^t m(\tau)\Omega(v(\tau))d\tau). \end{aligned}$$

Let

$$w(t) = v(t) + L \int_0^t m(\tau)\Omega(v(\tau))d\tau$$

Then $w(0) = v(0) = c, \qquad v(t) \leq w(t)$,
and

$$\begin{aligned} w'(t) &= v'(t) + Lm(t)\Omega(v(t)) \\ &\leq M_1 p(t)\Omega_0(w(t)) + Lm(t)\Omega(w(t)) \\ &\leq \hat{m}(t)[\Omega_0(w(t)) + \Omega(w(t))]. \end{aligned}$$

This implies

$$\int_{w(0)}^{w(t)} \frac{ds}{\Omega_0(s) + \Omega(s)} \leq \int_0^b \hat{m}(s)ds < \int_c^\infty \frac{ds}{\Omega_0(s) + \Omega(s)}, \qquad t \in J$$

This inequality implies that there is a constant K such that w(t) $\leq$K, t $\in$J and hence $\|x(t)\| \leq K, t \in J$, where K depends only on b and on the functions $\hat{m}, \Omega_0$ and Ω.

Second we must prove that the operator $F : C = C(J, X) \to C$ defined by

$$\begin{aligned} (Fx)(t) &= T(t)\phi(0) + \int_0^t T(t-\eta)BW^{-1}[x_1 - T(b)\phi(0) \\ &\quad - \int_0^b T(b-s)f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)ds](\eta)d\eta \\ &\quad + \int_0^t T(t-s)f(s, x(\sigma_1(s)), \int_0^s l(t,s)g(\tau, x(\sigma_2(\tau)))d\tau)ds \end{aligned}$$

is a completely continuous operator.
Let $B_k = \{x \in C, \quad \|x\| \leq k\}$ for some $k \geq 1$.
We first show that F maps B_k into an equicontinuous family. Let $x \in B_k$ and

$t_1, t_2 \in$ J. Then if $0 < t_1 < t_2 \leq b$

$$
\begin{aligned}
&\|(Fx)(t_1) - (Fx)(t_2)\| \\
&\leq \|T(t_1) - T(t_2)\|\|\phi(0)\| \\
&+ \|\int_0^{t_1} [T(t_1-\eta) - T(t_2-\eta)]BW^{-1}[x_1 - T(b)\phi(0) \\
&\quad - \int_0^b T(b-s)f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)ds](\eta)d\eta\| \\
&+ \|\int_{t_1}^{t_2} T(t_2-\eta)BW^{-1}[x_1 - T(b)\phi(0) \\
&\quad - \int_0^b T(b-s)f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)ds](\eta)d\eta\| \\
&+ \|\int_0^{t_1} [T(t_1-s) - T(t_2-s)]f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)ds\| \\
&+ \|\int_{t_1}^{t_2} T(t_2-s)f(s, x(\sigma_1(s)), \int_0^s l(s,\tau)g(\tau, x(\sigma_2(\tau)))d\tau)ds\| \\
&\leq \|T(t_1) - T(t_2)\|\|\phi(0)\| \\
&\quad + \int_0^{t_1} \|T(t_1-\eta) - T(t_2-\eta)\|M_2M_3[\|x_1\| + M_1\|\phi(0)\| \\
&\qquad + M_1 \int_0^b \alpha_k(s)ds]d\eta \\
&\quad + \int_{t_1}^{t_2} \|T(t_2-\eta)\|M_2M_3[\|x_1\| + M_1\|\phi(0)\| \\
&\qquad + M_1 \int_0^b \alpha_k(s)ds]d\eta \\
&\quad + \int_0^{t_1} \|[T(t_1-s) - T(t_2-s)]\|\alpha_k(s)ds \\
&\quad + \int_{t_1}^{t_2} \|T(t_2-s)\|\alpha_k(s)ds
\end{aligned}
$$

The right-hand side tends to zero as $t_2 - t_1 \to 0$ and ϵ sufficiently small, since the compactness of T(t) for t>0 implies the continuity in the uniform operator topology.

Thus F maps B_k into an equicontinuos family of functions. It is easy to see that the family FB_k is uniformly bounded.

Next we show $\overline{FB_k}$ is compact. Since we have shown FB_k is an equicontinuous collection, it suffices by the Arzela-Ascoli theorem to show that F maps B_k into a precompact set in X.

Let $0 < t \leq b$ be fixed and ϵ a real number satisfying $0 < \epsilon < t$. For $x \in B_k$ we define

$$(F_\epsilon x)(t) \;=\; T(t)\phi(0) + \int_0^{t-\epsilon} T(t-\eta)BW^{-1}[x_1 - T(b)\phi(0)$$

$$- \int_0^b T(b-s)f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)ds](\eta)d\eta$$
$$+ \int_0^{t-\epsilon} T(t-s)f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)ds$$
$$= T(t)\phi(0) + T(\epsilon)\int_0^{t-\epsilon} T(t-\eta-\epsilon)BW^{-1}[x_1 - T(b)\phi(0)$$
$$- \int_0^b T(b-s)f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)ds](\eta)d\eta$$
$$+ T(\epsilon)\int_0^{t-\epsilon} T(t-s-\epsilon)f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)ds$$

Since T(t) is a compact operator, the set $Y_\epsilon(t) = \{(F_\epsilon x)(t) : x \in B_k\}$ is precompact in X for every ϵ, $0 < \epsilon < t$. Moreover for every $x \in B_k$ we have

$$\begin{aligned}
\|(Fx)(t) - (F_\epsilon x)(t)\| \leq & \int_{t-\epsilon}^t \|T(t-\eta)BW^{-1}[x_1 - T(b)\phi(0) \\
& - \int_0^b T(b-s)f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)ds](\eta)\|d\eta \\
& + \int_{t-\epsilon}^t \|T(t-s)f(s,x(\sigma_1(s)),\int_0^s g(\tau,x(\sigma_2(\tau)))d\tau)\|ds \\
\leq & \int_{t-\epsilon}^t \|T(t-\eta)\| M_2M_3[\|x_1\| + M_1\|\phi(0)\| \\
& \quad + M_1\int_0^b \alpha_k(s)ds]d\eta \\
& + \int_{t-\epsilon}^t \|T(t-s)\|\alpha_k(s)ds
\end{aligned}$$

Therefore there are precompact sets arbitrarily close to the set $\{(Fx)(t) : x \in B_k\}$. Hence the set $\{(Fx)(t) : x \in B_k\}$ is precompact in X.

It remains to show that $F : C \to C$ is continuous. Let $\{x_n\}_0^\infty \subseteq C$ with $x_n \to x$ in C. Then there is an integer r such that $\|x_n(t)\| \leq r$ for all n and $t \in J$, so $x_n \in B_r$ and $x \in B_r$.
By (iv)

$$f(t,x_n(\sigma_1(t)),\int_0^t l(t,s)g(s,x_n(\sigma_2(s)))ds) \to f(t,x(\sigma_1(t)),\int_0^t l(t,s)g(s,x(\sigma_2(s)))ds)$$

for each $t \in J$ and since

$$\|f(t,x_n(\sigma_1(t)),\int_0^t l(t,s)g(s,x_n(\sigma_2(s)))ds) \to f(t,x(\sigma_1(t)),\int_0^t l(t,s)g(s,x(\sigma_2(s)))ds)\| \leq 2\alpha_r(t)$$

we have by dominated convergence

$$\|Fx_n - Fx\| = \sup_{t\in J}\|\int_0^t T(t-\eta)BW^{-1}[\int_0^b T(b-s)$$

$$[f(s,x_n(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x_n(\sigma_2(\tau)))d\tau)$$
$$-f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)]](\eta)d\eta$$
$$+\int_0^t T(t-s)[f(s,x_n(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x_n(\sigma_2(\tau)))d\tau)$$
$$-f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)]ds\|$$
$$\leq \int_0^b \|T(t-\eta)\| M_2 M_3$$
$$[M_1\int_0^b \|[f(s,x_n(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x_n(\sigma_2(\tau)))d\tau)$$
$$-f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)]]d\eta\|$$
$$+\int_0^b \|T(t-s)\|\|[f(s,x_n(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x_n(\sigma_2(\tau)))d\tau)$$
$$-f(s,x(\sigma_1(s)),\int_0^s l(s,\tau)g(\tau,x(\sigma_2(\tau)))d\tau)]\|ds \to 0$$

Thus F is continuous. This completes the proof that F is completely continuous.

Finally the set $\zeta(F)=\{x\in C: x=\lambda Fx, \lambda\in(0,1)\}$ is bounded, as we proved in the first step. Consequently by Schaefer's theorem the operator F has a fixed point in C. This means that any fixed point of F is a mild solution of (1) on J satisfying $(Fx)(t)=x(t)$. Thus the system (1) is controllable on J.

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