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THE MATHEMATICAL WORK OF MIRON NICOLESCU (1903-1975)

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This year, Professor Miron Nicolescu, one of the greatest Rumanian mathematicians and a long time leading personality of the Rumanian mathematical community, would have been 80 years old. He died 8 years ago, by a devastating heart attack, shortly after the forced closing of the major mathematical research institution in Rumania, to which, as head, he dedicated more than ten years of his life. As a mathematician, M. Nicolescu substantially contributed to the study of several classes of functions defined by iterated classical partial differential equations. As a scientific leader he was highly effective in the formation and the promotion of many mathematicians in Rumania (and I gratefully acknowledge to be one of these mathematicians). The 80th anniversary of the birth of M. Nicolescu is therefore a good opportunity to recall the main contributions of his scientific work.

Firstly, here is a curriculum vitae of Miron Nicolescu. He was born on August 27, 1903 (in Giurgiu, Rumania). He graduated in mathematics from the University of Bucharest in 1924 and the famous École Normale Supérieure of Paris in 1927, got his Doctorat d'État at Sorbonne in 1928, and was appointed associate professor the same year at the University of Cernăuţi (Rumania; the town is now in the Soviet Union and the university is closed), which had, in the inter bellum period, the best department of mathematics in Rumania. In 1933 he was promoted to full professor at that university, and in 1936 he was elected a correspondent member of the Science Academy of Rumania. In 1940 he moved to the University of Bucharest, where he shortly became the best professor of mathematics that university ever had. M. Nicolescu became the director of the research institute of mathematics, referred above, in 1963, and the president of the National Academy of Rumania in 1966. Through that office he was coopted by the governmental establishment in Rumania, but M. Micolescu viewed himself, and acted accordingly, as a defensive buffer (unhappily often more as a lightening rod) for the mathematical community of Rumania. Actually, he did so even in the 1950's, saving the scientific career of several of his students, although he too was then a political outcast.

The mathematical research activity of Miron Nicolescu extended in several directions and includes over 120 scientific papers. The main directions (in chronological order) are the following.

(a) The theory of areolar derivative (1927-1930)

(b) The theory of polyharmonic functions (1930-1970)

(c) The theory of polycaloric functions (1932-1966)

(d) Hyperbolic analysis (1938-1956)

(e) Analyticity with respect to an operator (1956-1963).

The positions of these directions in the overall contemporary mathematical map are, of course, different. However, in all of them the contribution of M. Nicolescu, even if sometimes overlooked, is organically interwoven, in their present state, as chapters of mathematical knowledge or investigation. Here is a brief and impressionistic presentation of these contributions.

(a) The areolar derivative is presently denoted by $\frac{\partial}{\partial \bar{z}}$. The terminology was introduced in [P3] by D. Pompeiu (one of the leading Rumanian mathematicians in the first quarter of this century) because for C^1-functions f on a domain $\Omega \subset \mathbb{C}$, one has

$$\frac{\partial f}{\partial \bar{z}}(\omega) = \lim_{D} \frac{1}{2i \text{ area of } D} \int_{\partial D} f(z)\, dz \,,$$

where the limit is taken for simply connected domains D, $\omega \in D \subset \Omega$, such that the length of $\partial D \to 0$. D. Pompeiu was also the first to study the equation $\frac{\partial f}{\partial \bar{z}} = g$ with discontinuous g and to emphasize the role of its primitive

$$\frac{1}{2i} \iint_D \frac{g(s+it)}{s+it-\omega}\, ds\, dt$$

in the study of analytic functions with large singular sets [P2]. In 1927, in [N1], M. Nicolescu considered the maps $(x,y) \longmapsto (u,v)$ from a domain $\Omega \subset \mathbb{C}^2$ to $\mathbb{C}^2$ satisfying the areolar Cauchy-Riemann equations

$$(1) \qquad \frac{\partial u}{\partial \bar{x}} = \frac{\overline{\partial v}}{\partial \bar{y}} , \qquad \frac{\partial u}{\partial \bar{y}} = - \frac{\overline{\partial v}}{\partial \bar{x}} ,$$

which, by writing $x = x_1 + ix_2$, $y = x_3 + ix_4$, $u = u_1 + iu_2$, and $v = u_3 - iu_4$ with $x_1,\dots,u_4$ real, are equivalent to

$$(2) \qquad \begin{aligned}
&\frac{\partial u_1}{\partial x_1} - \frac{\partial u_2}{\partial x_2} - \frac{\partial u_3}{\partial x_3} - \frac{\partial u_4}{\partial x_4} = 0 , \\
&\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} + \frac{\partial u_3}{\partial x_4} - \frac{\partial u_4}{\partial x_3} = 0 , \\
&\frac{\partial u_1}{\partial x_3} - \frac{\partial u_2}{\partial x_4} + \frac{\partial u_3}{\partial x_1} + \frac{\partial u_4}{\partial x_2} = 0 , \\
&\frac{\partial u_1}{\partial x_4} + \frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2} + \frac{\partial u_4}{\partial x_1} = 0 .
\end{aligned}$$

The equations (1) were studied by M. Nicolescu in his thesis [N2] by exploiting the analogy with the study of analytic functions based on the classical Cauchy-Riemann system. The system (2) was rediscovered independently by G. Moisil-N. Teodorescu [M2] and by R. Fueter [F3] as basic in the theory of monogeneous maps of quaternions. Recently this theory became relevant in the gauge fields theory [G2].

(b) A polyharmonic function (of order $\le p$) is a solution u of the equation $\Delta^p u = 0$ on some domain $\Omega \subset \mathbb{R}^n$, where Δ is the Laplace operator. This terminology, today universally used, was introduced by M. Nicolescu in his papers, starting with the note [N10] and the memoir [N11]. Polyharmonic functions (especially of order 2) occur often in continuum mechanics. The classical theory of the polyharmonic functions was essentially completed by M. Nicolescu in his papers [N3-N5], [N7], [N10-N12] and his monograph [N13] written, at the suggestion of Paul Montel, for the highly respected series of monographic works "Actualites Scientifiques et Industrielles" published by P. Montel in Paris. This short monograph remained for a long time the basic reference for polyharmonic functions (see for instance [B1], p. 44). Nicolescu's principal tool in extending many classical theorems from harmonic functions to polyharmonic functions was his elegant weighted mean theorem [N7]:

For a locally integrable function u on a domain $\Omega \subset \mathbb{R}^n$ define the iterated means $\mu_s(u,r)$ by

$$\mu_0(u,r)(x_0) = \text{mean of } u \text{ on } \{x: |x - x_0| = r\}$$

(where $r < r(x_0) = \mathrm{dist}(x_0, \partial\Omega)$) and

$$\mu_s(u,r) = \frac{n}{r^n}\int_0^r \rho^{n-1}\,\mu_{s-1}(u,\rho)\,d\rho \quad (n = 1,2,\ldots) .$$

Then u is polyharmonic (of order $\leq p$) if and only if

$$(3) \qquad \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ a_{n0} & a_{n1} & a_{n2} & \cdots & a_{n,p-1} \\ a_{n0}^2 & a_{n1}^2 & a_{n2}^2 & \cdots & a_{n,p-1}^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{np}^{p-1} & a_{n1}^{p-1} & a_{n2}^{p-1} & \cdots & a_{n,p-1}^{p-1} \end{vmatrix} u(x) = \begin{vmatrix} \mu_0 & 1 & 1 & \cdots & 1 \\ \mu_1 & a_{n1} & a_{n2} & \cdots & a_{n,p-1} \\ \mu_2 & a_{n1}^2 & a_{n2}^2 & \cdots & a_{n,p-1}^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mu_{p-1} & a_{n1}^{p-1} & a_{n2}^{p-1} & \cdots & a_{n,p-1}^{p-1} \end{vmatrix}$$

for all $x \in \Omega$ and all $r \in (0,r(x))$ sufficiently small, where

$$a_{ns} = n(n + 2s)^{-1} , \qquad \mu_s = \mu_s(u,r)(x) \qquad (s = 0,1,2,\ldots) .$$

The appreciation of M. Nicolescu's contributions on polyharmonic functions is clearly illustrated by Mauro Picone's comments ([P1], p. 287) on Nicolescu's extension to polyharmonic functions of the second Harnack convergence theorem [N11]. An important by-product obtained during this extension was the discovery of the natural concept of complete subharmonicity of order p, which allowed M. Nicolescu to extend to this new concept many of the properties related to subharmonicity [N11], [N12].

M. Nicolescu also initiated the fine study of singularities of polyharmonic functions [N17], [N18] (results which were later improved by G. Fichera [F1]), as well as the study of almost periodic polyharmonic functions on a half-space [N23].

(c) A polycaloric function (of order $\leq p$) is a solution u of the equation $\bigcirc^p u = 0$ in a domain $\Omega \subset \mathbb{R}^n$ (for the space variables) $\times \mathbb{R}$ (for the time variable), where $\bigcirc$ is the operator $\frac{\partial}{\partial t} - \Delta$. This is the terminology and the notation introduced by M. Nicolescu in his memoir [N24] of 1954, which constitutes even today the most exhaustive study of polycaloric functions. However, his first contribution to these topics came in 1932, where he proved a Liouville type theorem for the heat equation $\bigcirc u = 0$ [N6]. Later, in 1935, he independently discovered the uniqueness theorem for the heat equation which is presently referred to as Tihonov's Theorem. His approach [N14] was based on an original, remarkable representation formula for a solution $u = u(x,t)$ of the heat equation (with one space variable) in a rectangle $[r,R] \times [h,y]$:

$$(4) \qquad u(x,y) = \frac{1}{2\sqrt{\pi(y-h)}} \int_r^R \left[e^{-\frac{(x-\xi)^2}{4(y-\eta)}} - e^{-\frac{(2R-x-\xi)^2}{4(y-\eta)}} \right] u(\xi,h)\, d\xi -$$

$$- \frac{1}{2\sqrt{\pi}} \int_h^y \frac{e^{-\frac{(x-\xi)^2}{4(y-\eta)}}}{\sqrt{y-\eta}} \left[\left(\frac{\partial u}{\partial \xi}\right)_{\xi=r} - \frac{(x-r)u(x,\eta)}{2(y-\eta)} \right] d\eta +$$

$$+ \frac{1}{2\sqrt{\pi}} \int_h^y \frac{e^{-\frac{(2R-r-x)^2}{4(y-\eta)}}}{\sqrt{y-\eta}} \left[\left(\frac{\partial u}{\partial \xi}\right)_{\xi=r} - \frac{(2R-r-x)u(r,\eta)}{2(y-\eta)} \right] d\eta -$$

$$- \frac{1}{2\sqrt{\pi}} \int_h^y e^{-\frac{(x-R)^2}{4(y-\eta)}} \frac{(x-R)u(R,\eta)}{(y-\eta)^{3/2}}\, d\eta .$$

The extension of this formula to n space variables is due to P. Mustaţă [M3]. It turned out, later on, that a slight change of the proof in [N14] yields the following stronger uniqueness theorem [N30] (see also [F2], [K1], or [J1]):

The initial value problem for the heat equation in $\mathbb{R} \times [0,\delta]$ has a unique solution in the class of functions satisfying the condition

$$\int_0^\delta dt \int_{-\infty}^{\infty} e^{-\kappa x^2} \max\{0, -u(x,t)\}\, dx < \infty ,$$

where κ is a suitable constant > 0 depending on u.

Returning to the memoir [N24], one should notice that, on one hand, it is conceived as a monograph of classical analysis in which the derivative $\frac{d}{dt}$ is replaced by the operator $\bigcirc$ and that, on the other hand, the techniques used are partially modelled on M. Nicolescu's previous treatment of polyharmonic functions. This last fact is well illustrated by the basic role played in [N24] by the following weighted mean theorem:

For a locally integrable function u on a band $\mathbb{R} \times (0,\delta)$, define the iterated means $\mu_s(u,h)$ by

$$\mu_0(u,h)(x,t) = \frac{1}{2\sqrt{\pi h}} \int_{-\infty}^{\infty} e^{-\frac{(x-\xi)^2}{4h}} u(\xi, t-h)\, d\xi$$

(where $0 < h < t < \delta$) and

$$\mu_s(u,h) = \frac{2}{h^2} \int_0^h k\mu_{s-1}(u,k)\, dk \qquad (s = 1,2,\dots).$$

Then u is polycaloric (of order $\le p$) if and only if

$$\text{(5)} \qquad \begin{vmatrix} 1 & 1 & \dots & 1 \\ 1 & 2/3 & \dots & 2/(p+1) \\ \dots & \dots & \dots & \dots \\ 1 & (2/3)^{p-1} & \dots & (2/(p+1))^{p-1} \end{vmatrix} u(x,t) = \begin{vmatrix} \mu_0 & 1 & \dots & 1 \\ \mu_1 & 2/3 & \dots & 2/(p+1) \\ \dots & \dots & \dots & \dots \\ \mu_{p-1} & (2/3)^{p-1} & \dots & (2/(p+1))^{p-1} \end{vmatrix}$$

for all $-\infty < x < \infty$, $0 < h < t < \delta$, where

$$\mu_s = \mu_s(u,h)(x,t) \qquad (s = 0,1,2,\dots).$$

(d) The bidimensional or "hyperbolic" derivative $Du(x,y)$ of a function u defined on a domain $\Omega \subset \mathbb{R}^2$ is, by definition

$$Du(x,y) = \lim_{h,k\to 0} \frac{u(x+k, y+k) - u(x+h, y) - u(x, y+k) + u(x,y)}{hk}$$

whenever the limit exists. Using this definition, K. Bögel [B3] initiated the analysis of functions of two variables based on D in analogy with the analysis of functions of one variable based on the derivative $\frac{d}{dx}$. The development of

K. Bögel's program is in great part due to M. Nicolescu [N15], [N16], [N21], [N22]. Indeed, with the monographic work [N21], the bases of the hyperbolic analysis are completed in analogy with those of the calculus. Here is an illustrative theorem of [N21]:

If for u on Ω all hyperbolic derivatives $D^n u$ $(n = 1,2,3,\ldots)$ exist and

$$\sup_n \sup_\Omega |D^n u| < \infty ,$$

then u is hyperbolically analytic in Ω, that is, for any $(x_0,y_0) \in \Omega$, one has the expansion

$$u(x,y) = \sum_{n=0}^{\infty} (x - x_0)^n (y - y_0)^n \phi_n(x,y) ,$$

where the convergence is uniform in a neighborhood Ω_0 of (x_0,y_0) and the functions ϕ_n on Ω_0 are D-constant (i.e., $D\phi_n = 0$, $n = 1,2,\ldots$); (notice that the function u may not even be continuous in the classical sense). This concept of analyticity was shown to be useful earlier by N. Cioranescu [C1].

(e) The preceding result suggested to M. Nicolescu the following notion of analyticity with respect to a linear operator A on a commutative normed algebra $\mathcal{B}$ with unity e [N28], [N29]: An element $a \in \mathcal{B}$ is called A-analytic if $a = \sum_{n=0}^{\infty} t^n a_n$, where the series strongly converges in $\mathcal{B}$ and

$$Aa_n = 0 \quad (n = 0,1,2,\ldots) , \quad At = e .$$

He showed that if $\mathcal{B}$ is an appropriate algebra of bounded functions and if $A = D$, $\bigcirc$ or Δ, then

$$\sup_n \sup_x |A^n u(a)| < \infty$$

implies that u is A-analytic [N21], [N25], [N26]. He also developed an axiomatic framework for A-analyticity which includes the three particular cases mentioned above. In particular, in M. Nicolescu's abstract framework, the condition

$$\sup_n \| A^n a \| < \infty$$

also implies that a is A-analytic [N28], [N29]. A subsequent similar but more general development of this notion of A-analyticity is due to R. Bittner [B2].

But M. Nicolescu's work is not restricted to the directions (a)-(e). He also published interesting research papers in other topics of classical and modern mathematical analysis (ordinary differential equations, double series and sequences, measure theory, functional analysis, etc.). For instance in measure theory, M. Nicolescu has two pioneering papers [N8], [N9] on the Lebesgue type theory of the Riemann integral. Here is one of his results on this subject [N9]:

Assume that for a bounded function f on $[a,b]$ at least one of the sets

$$\{x: f(x) > \alpha\} , \quad \{x: f(x) \geq \alpha\}$$

is Jordan measurable for all α. (A set $X \subset \mathbb{R}$ is Jordan measurable if its boundary is of measure = 0.) Then f is Riemann integrable on $[a,b]$.

For the modern development of this theory we refer to S. Marcus [M1].

This brief survey of the mathematical work of Miron Nicolescu cannot be considered complete without mentioning his treatise on mathematical analysis [N31]. This treatise is a textbook for several courses of classical and modern analysis as well as a coherent collection of monographs on basic topics of real analysis, each subject presented in the largest functional framework in which the richness of the classical framework can still be preserved. Even now, more than 20 years after its publication, this treatise does not seem to have been superseded in its scope and its realization. It is regrettable that the treatise was never translated from Rumanian to an internationally known language, although its reviewers recommended this (see, for instance, [G1]).

There is no doubt that Miron Nicolescu is alive in the heart of his students, in the culture of his country, and in the mathematics of the world.

REFERENCES

G. Birkhoff

[B1] Hydrodynamics. A study in logic, fact and similitude. Princeton Univ. Press (Princeton, 1960), 183 pp.

R. Bittner

[B2] Operational calculus in linear spaces, Studia Math., 20 (1961), 1-18.

K. Bögel

[B3] Mehrdimensionale Differentiation von Funktionen mehrer Veränderlichen, J. Reine Angen. Math., 170 (1934), 197-217.

N. Cioranescu

[C1] Sur les fonctions de deux variables réelles douées d'analyticité hyperbolique et leurs applications, Mathematica, 16 (1938), 180-190.

G. Fichera

[F1] Sviluppi in serie e teoremi di decomposizione in somma per le funzioni iperharmoniche, Rend. Circ. Nat. Palermo, 63 (1941), 24 pp.

A. Friedmann

[F2] Partial differential equations of parabolic type. Prentice Hall (Englewood Cliffs, 1964), 347 pp.

R. Fueter

[F3] Die Funktionentheorie der Differentialgleichungen $\Delta u = 0$ und $\Delta\Delta u = 0$ mit vier reellen Variablen, Comment. Math. Helv., 7 (1934/35), 307-330.

S.A. Gaczályi

[G1] Review of "M. Nicolescu, Analiza Matematica, I-II-III", Publicationes Math., 9 (1962), 356-357.

F. Gürsey

[G2] Quaternion methods in fields theory, Proc. John Hopkins Workshop on Current Problems in Particle Theory #4 (1980), John Hopkins University (Baltimore, 1980), 255-288.

R. Johnson

[J1] Representation theorems and Fatou theorems for second-order linear parabolic partial differential equations, Proc. London Math. Soc., Ser. 3, 23 (1971), 325-347.

M. Kryzański

[K1] Solutions of a linear second order equation of parabolic type defined in an unbounded domain, Differential Equations and Their Applications Proc. Symp. (Prague, 1962), Academic Press (New York, 1963), 55-63.

S. Marcus

[M1] On a Lebesgue type theory for the Riemann integral (in Rumanian), Studii Cerc. Mat., 9 (1958), 333-369.

G. Moisil and N. Teodorescu

[M2] Fonctions holomorphes dans l'espace, Mathematica, 5 (1931), 142-159.

P. Mustaţă

[M3] Sur la representation de Poisson des solutions de problème de Cauchy pour l'équation de la chaleur, Rev. Roem. Math. Pures Appl., 11 (1966), 673-691.

M. Nicolescu

[N1] Sur les fonctions de bipoint et les fonctions aréolairement conjugées, C.R. Acad. Sci. Paris, 185 (1927), 142-144.

[N2] Fonctions complexes dans le plan et dans l'espace, Thèse #1987. Faculté des Sciences de Paris, Mathematiques, Sorbonne 1928, Gauthier-Villars (Paris, 1928), 91 pp.

[N3] Extension du théorème de Gauss aux fonctions harmoniques d'ordre p , C.R. Acad. Paris, 191 (1930), 515-517.

[N4] Sur les fonctions harmoniques d'ordre p , Bull. Soc. Math. France, 59 (1931), 75-87.

[N5] Sur les fonctions harmoniques et sous-harmoniques d'ordre p , C.R. Acad. Sci. Paris, 1152-1154.

[N6] Extension du théorème de Liouville-Picard a l'équation de Fourier, Congrès Intern. de Math. (Zürich, 1932), p. 329.

[N7] Sur les fonctions de n variables, harmoniques d'ordre p , Bull. Soc. Math. France, 60 (1932), 129-151.

[N8] Sur quelques proprietés élementaires de le mesure au sense de Jordan, Bull. Fac. St. Cernauti, 6 (1932), 222-224.

[N9] Sur les fonctions mesurables(J), Bull. Sci. Math., II Ser., 57 (1933) 276-281.

[N10] Représentation des fonctions continues de plusieurs variables par des series uniformément convergentes de fonctions polyharmoniques, C.R. Acad. Paris, 199 (1934), 114-116.

[N11] Recherches sur les fonctions polyharmoniques, Ann. Scient. Ecole Norm. Sup., III Ser., 57 (1935), 183-220.

[N12] Familles normales de fonctions polyharmoniques, Bull. Soc. Roum. Sci., 38 (1936), 49-52.

[N13] Les fonctions polyharmoniques, Actualités Scientifiques et Industrielles #331, Hermann (Paris, 1936), 54 pp.

[N14] Sur l'equation de la chaleur, Coment. Math. Helv., 10 (1937), 3-17.

[N15] Sur quelques propositions d'analyse infinitésimale pluridimensionnelle, C.R. Acad. Sci. Roum., 2 (1937-1938), 228-231.

[N16] Continuité et dérivation polydimensionnelle et laplacienne des suites, Rev. Math. Union Interbalk., 30 (1940), 1-16.

[N17] Nouvelles recherches sur les fonctions polyharmoniques. I, Disq. Math. Phys., 1 (1940-1941), 43-56; II, idem, 173-187.

[N18] Remarques sur mon mémoir: Recherches sur les fonctions polyharmoniques Disq. Math. Phys., 1 (1940-1941), 189-190.

[N19] Sur le théorème fondamental de convergence de Pringsheim pour les suites doubles, Bull. Math. Soc. Roum. Sci., 47 (1945-1946), 3-9.

[N20] On the compactness criterium of A. Kolmogorov (in Rumanian), Bull. Stiint. Acad. R.P. Romane, Sect. Mat. Fiz. Chim., 2 (1950), 407-415.

[N21] Contributions to a hyperbolic analysis on the plane (in Rumanian), Studii Cerc. Mat., 3 (1952), 7-51.

[N22] The polydimensional derivatives of different orders (in Rumanian), Com. Acad. R.P. Romane, 2 (1952), 181-184.

[N23] Almost periodic polyharmonic functions (in Rumanian), Bull. Stiint. Acad. R.P. Romane, Sect. Mat. Fiz. Chim., 5 (1953), 273-283.

[N24] The iterated heat equation (in Rumanian), Studii Cerc. Mat., 5 (1954), 243-332.

[N25] La structure des solutions des équations aux dérivées partielles du type elliptiques ou du type parabolique, Magyar Tud. Akad. Nat. Kutató Int. Közl., 1 (1956), 465-479.

[N26] Sur quelques problèmes liés a l'opérateur itéré de la chaleur, Communication at the IVth Congress of the Rumanian Mathematicians (Bucharest, mai 1956).

[N27] Le problème de l'analyticité des fonctions réelles, Rev. Math. Pures Appl., 2 (1957), 53-59.

[N28] Problème de l'analyticité par rapport à un opérateur linéaire, Studia Math., 16 (1958), 353-363.

[N29] Contributions à l'analyse attachée à un opérateur linéaire dans une algèbre normée, Seminari 1962/63 Anal. Alg. Geom. et Top., 1, 1st; No. 2, Alta Math., Ediz. Cremonese (Rome, 1965), 37-56.

[N30] Sur l'unicité du problème de Cauchy pour l'équation de la chaleur, Rend. Acad. Naz. Lincei, 40 (1966), 785-792 (with C. Foiaş).

[N31] Mathematical Analysis (in Rumanian). Vol. I, Ed. Technica (Bucharest, 1957), 339 pp.; Vol. II, idem (1958), 535 pp.; Vol. III, ibidem (1960), 300 pp.

M. Picone

[P1] Nuovi indirizzi di ricerca nella teoria e nel calcolo delle soluzioni di talune equazioni lineari alle derivate partiali della fizico-matematice, Ann. Sc. Norm. Sup. Pisa, Ser. II, 5 (1936), 215-288.

D. Pompeiu

[P2] Sur la continuité des fonctions de variable complexe, Ann. Fac. Sci. Toulouse, 7 (1905), 265-315.

[P3] Sur une classe de fonctions d'une variable complexe, Rend. Circ. Mat. Palermo, 33 (1912), 108-113.