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Econometric Analysis on Efficiency of Estimators

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Abstract. This paper investigates the efficiency of an alternative to ratio estimator under the super population model with uncorrelated errors and a gamma-distributed auxiliary variable. Comparisons with usual ratio and unbiased estimators are also made.

Keywords: Bias, mean square error, ratio estimator super population.

1 Introduction

It is well known that the ratio method of estimation occupies an important place in sample surveys. When the study variate y and the auxiliary variate x is positively (high) correlated, the ratio method of estimation is quite effective in estimating the population mean of the study variate y utilizing the information on auxiliary variate x.

Consider a finite population with N units and let x_i and y_i denote the values for two positively correlated variates x and y respectively for the ith unit in this population, $i = 1, 2, \dots, N$. Assume that the population mean $\bar{X}$ of x is known. Let $\bar{x}$ and $\bar{y}$ be the sample means of x and y respectively based on a simple random sample of size n ($n < N$) units drawn without replacement scheme. Then the classical ratio estimator for $\bar{Y}$ is defined by

$$\bar{y}_r = \bar{y}(\bar{X}/\bar{x}). \tag{1}$$

The bias and mean square error (MSE) of $\bar{y}_r$ are, up to second order moments,

$$B(\bar{y}_r) = \lambda(R\,S_x^2 + S_{yx})/\bar{X} \tag{2}$$

$$M(\bar{y}_r) = \lambda(S_y^2 + R^2 S_x^2 - 2RS_{yx}), \tag{3}$$

where $\lambda = (N-n)/(nN)$, $R = \bar{Y}/\bar{X}$, $S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$ and

$S_{yx} = (N-1)^{-1}\sum_{n=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})$.

It is clear from (3) that $M(\bar{y}_r)$ will be minimum when

$$R = S_{yx}/S_x^2 = \beta, \tag{4}$$

where β is the regression coefficient of y on x. Also for $R = \beta$ the bias of $\bar{y}_r$ in (2) is zero. That is, $\bar{y}_r$ is almost unbiased for $\bar{Y}$.

Let $E(\bar{y}|\bar{x}) = \alpha + \beta\bar{x}$ be the line of regression of $\bar{y}$ on $\bar{x}$, where E denotes averaging over all possible sample design simple random sampling without replacement (SRSWOR). Then $\beta = S_{yx}/S_x^2$ and $\bar{Y} = \alpha + \beta\bar{X}$ so that, in general,

$$R = (\alpha/\bar{X}) + \beta \tag{5}$$

It is obvious from (4) and (5) that any transformation that brings the ratio of population means closer to β will be helpful in reducing the mean square error (MSE) as well as the bias of the ratio estimator $\bar{y}_r$. This led Srivenkataramana and Tracy (1986) to suggest an alternative to ratio estimator $\bar{y}_r$ as

$$\bar{y}_a = \bar{z}\left(\bar{X}/\bar{x}\right) + A = \bar{y}_r - A\left\{\left(\bar{X}/\bar{x}\right) - 1\right\} \tag{6}$$

which is based on the transformation

$$\bar{z} = \bar{y} - A \tag{7}$$

where $E(\bar{z}) = \bar{Z}(= \bar{Y} - A)$ and A is a suitably chosen scalar.

In this paper exact expressions of bias and MSE of $\bar{y}_a$ are worked out under a super population model and compared with the usual ratio estimator.

2 The super population model

Following Durbin (1959) and Rao (1968) it is assumed that the finite population under consideration is itself a random sample from a super population and the relation between x and y is of the form:

$$y_i = \alpha + \beta x_i + u_i; \quad (i = 1, 2, \ldots, N)$$

where α and β are unknown real constants; u_i's are uncorrelated random errors with conditional (given x_i) expectations

$$E\left(u_i\,|x_i\right) = 0$$
$$E\left(u_i^2\,|x_i\right) = \delta x_i^g$$

$(i = 1, 2, \ldots, N)$, $0 < \delta < \infty$, $0 \leq g \leq 2$ and x_i are independently identically distributed (i.i.d.) with a common gamma density

$$G(\theta) = e^{-x}x^{\theta-1}/\Gamma\theta, \quad x > \theta,\ 2 < \theta < \infty \tag{8}$$

We will write E_x to denote expectation operator with respect to the common distribution of x_i $(i = 1, 2, 3, \ldots, N)$ and $E_x\ E_c$, as the over all expectation operator for the model. We denote a design by p and the design expectation E_p, for instance, see Chaudhuri and Adhikary (1983,89) and Shah and Gupta (1987). Let 's' denote a simple random sample of N distinct labels chosen without replacement out of $i = 1, 2, 3, \ldots, N$. Then

$$X(N = \bar{X}) = \sum_{i \in s} x_i + \sum_{i \notin s} X_i$$

Following Rao and Webster (1966) we will utilize the distributional properties of x_j/x_I, $\sum\limits_{i \in s} x_i$, $\sum\limits_{i \notin s} x_i$, $\sum\limits_{i \in s} x_i / \sum\limits_{i \notin s} x_i$ in our subsequent derivations.

3 The bias and mean square error

The estimator $\bar{y}_a$ in (6) can be written as

$$\bar{y}_a = \left[(1/n) \left(\sum_{i \in s} y_i \right) \frac{\left(n \sum_{i=1}^{N} x_i \right)}{\left(N \sum_{i \in s} x_i \right)} - A \left\{ \frac{\left(n \sum_{i=1}^{N} x_i \right)}{\left(N \sum_{i \in s} x_i \right)} - 1 \right\} \right] \tag{9}$$

based on a simple random sample of n distinct labels chosen without replacement out of $i = 1, 2, \ldots, N$.

The bias

$$B = E_p(\bar{y}_a - \bar{Y}) \tag{10}$$

of $\bar{y}_a$ has model expectation $E_m(B)$ which works out as follows:

$$\begin{aligned}
&E_m\left(B(\bar{y}_a)\right) \\
&= E_p E_x E_c \left[\left\{ \alpha + \beta (1/n) \left(\sum_{i \in s} x_i \right) + \bar{u} \right\} \frac{n \sum_{i=1}^{N} x_i}{n \sum_{i \in s} x_i} - A \left\{ \frac{n \left(\sum_{i=1}^{N} x_i \right) - 1}{N \left(\sum_{i \in s} x_i \right)} \right\} \right] \\
&\quad - E_x E_c (\alpha + \beta \bar{X}) \\
&= E_p E_x E_c \left[\alpha \left(n \sum_{i=1}^{N} x_i / N \sum_{i \in s} x_i \right) + \beta (1/N) \left(\sum_{i=1}^{N} x_i \right) + \left(\sum_{i \in s} u_i \right) \left(\sum_{i=1}^{N} x_i / N \sum_{i \in s} x_i \right) \right. \\
&\quad \left. - A \left\{ \left(\left(n \sum_{i=1}^{N} x_i \right) \Big/ \left(N \sum_{i \in s} x_i \right) \right) - 1 \right\} \right] - E_x E_c (\alpha + \beta \bar{X}) \\
&= E_p E_x \left[\alpha \left(n \sum_{i=1}^{N} x_i \right) / \left(N \sum_{i \in s} x_i \right) + \beta \bar{X} - A \left\{ \left(n \sum_{i=1}^{N} x_i \right) / \left(N \sum_{i \in s} x_i \right) - 1 \right\} \right] \\
&\quad - \alpha - \beta E_x(\hat{X}) \\
&= E_x \left[\alpha (n/N) \left(1 + \sum_{i \notin s} x_i / \sum_{i \in s} x_i \right) - A \left\{ (n/N) \left(1 + \sum_{i \notin s} x_i / \sum_{i \in s} x_i \right) - 1 \right\} \right] - \alpha \\
&= \alpha (n/N) \left\{ 1 + (N-n) \theta / (n\theta - 1) \right\} - A \left\{ (n/N) \left(1 + (N-n) \theta / (n\theta - 1) \right) - 1 \right\} - \alpha \\
&= \alpha \left[(n/N - 1) + \left\{ n (N-n) \theta / N (n\theta - 1) \right\} \right] \\
&\quad - A \left[-(N-n)/N + \left\{ (N-n) n\theta / N (n\theta - 1) \right\} \right] \\
&= (N-n)(\alpha - A) / N (n\theta - 1)
\end{aligned} \tag{11}$$

For SRSWOR sampling scheme, the mean square error

$$M(\bar{y}_a) = E_p \left(\bar{y}_a - \bar{Y} \right)^2 \tag{12}$$

of $\bar{y}_a$ has the following formula for model expectations $E_m(M\bar{y}_a)$:

$$E_m\left(M\left(\bar{y}_a\right)\right) = \left[E_m\left(M\left(\bar{y}_r\right)\right) + \frac{\left(N-n\right)\left(Nn\theta+2N-2n\right)\left(A^2-2A\alpha\right)}{N^2\left(n\theta-1\right)\left(n\theta-2\right)}\right] \tag{13}$$

where

$$M\left(\bar{y}_r\right) = E_p\left(\bar{y}_r - \bar{Y}\right)^2 \tag{14}$$

is the MSE of $\bar{y}_r$ under SRSWOR scheme has the model expectation

$$E_m\left(M\left(\bar{y}_r\right)\right) = \left\{\left(N-n\right)/N^2\right\}\left[\left\{\frac{\left(Nn\theta+2N-2n\right)\alpha^2}{\left(n\theta-1\right)\left(n\theta-2\right)}\right\} + \frac{\delta\left\{\left(n\theta+g-1\right)\left(n\theta+g-2\right)+n\theta\left(N\theta-n\theta+1\right)\right\}}{\left(n\theta+g-1\right)\left(n\theta+g-2\right)}\frac{\Gamma\left(\theta+g\right)}{\Gamma\theta}\right] \tag{15}$$

[See, Rao (1968, p.439)].

Further, we note that for SRSWOR sampling scheme, the bias

$$B\left(\bar{y}_r\right) = E_p\left(\bar{y}_r - \bar{Y}\right) \tag{16}$$

of usual ratio estimator has the model expectation

$$E_m\left(B\left(\bar{y}_r\right)\right) = \left(N-n\right)\alpha/\left(n\theta-1\right) \tag{17}$$

We note from (11) and (17) that

$$\left|E_m\left(B\left(\bar{y}_a\right)\right)\right| < \left|E_m\left(B\left(\bar{y}_r\right)\right)\right|$$

if

$$\left|\left(\alpha - A\right)\right| < \left|\alpha\right|$$

or if

$$\left(\alpha - A\right)^2 < \alpha^2$$

or if

$$0 \langle A \langle 2\alpha \tag{18}$$

Further we have from (13) that

$$E_m\left(M\left(\bar{y}_a\right)\right) - E_m\left(M\left(\bar{y}_r\right)\right) < 0$$

if

$$\left(A^2 - 2A\alpha\right) < 0$$

or if

$$0 < A < 2\alpha \tag{19}$$

which is the same as in (18). Thus we state the following theorem:

Theorem 3.1. *The estimator $\bar{y}_a$ is less biased as well as more efficient than usual ratio estimator $\bar{y}_r$ if*

$$0 < A < 2\alpha \neq 0$$

i.e. when A lies between 0 and 2α. Therefore, when intercept term α ($\neq 0$) in the model (8) is sizable, there will be sufficient flexibility in picking A.

It is to be noted that for $\alpha = 0$, $\bar{y}_r$ is unbiased and efficient than $\bar{y}_a$. The minimization of (13) with respect to A leads to

$$A = \alpha = A_{opt}(\text{say}) \tag{20}$$

Substitution of (20) in (13) yields the minimum value of $E_m\left(M\left(\bar{y}_a\right)\right)$ as

$$\min .E_m\left(M\left(\bar{y}_a\right)\right) = \frac{(N-1)}{N^2}\frac{\delta\left[(n\theta+g-1)(n\theta+g-2)+n\theta(N\theta-n\theta+1)\right]}{(n\theta+g-1)(n\theta+g-2)}\frac{\Gamma(\theta+g)}{\Gamma\theta} \tag{21}$$

which equals to $E_m\left(M\left(\bar{y}_r\right)\right)$ when $\alpha = 0$. It is interesting to note that when $A = \alpha$, $\hat{y}_a$ is unbiased and attained its minimum average MSE in model (8). In practice the value of α will have to be assessed, at the estimation stage, to be used as A. To assess α, we may use scatter diagram of y versus x for data from a pilot study, or a part of the data from the actual study and judge the y-intercept of the best fitting line.

From (15) and (21) we have

$$\begin{aligned} &E_m\left(M\left(\bar{y}_r\right)\right) - \min .E_m\left(M\left(\bar{y}_a\right)\right) \\ &\quad = \left\{(N-n)(Nn\theta+2N-2n)\alpha^2\right\}/\left\{N^2(n\theta-1)(n\theta-2)\right\} > 0 \end{aligned} \tag{22}$$

which shows that $\bar{y}_a$ is more efficient than ratio estimator when $A = \alpha$ is known exactly. For $\alpha = 0$

$$\min .E_m\left(M\left(\bar{y}_a\right)\right) = E_m\left(M\left(\bar{y}_r\right)\right) \tag{23}$$

For SRSWOR, the variance

$$V(\bar{y}) = E_p\left(\bar{y} - \bar{Y}\right)^2 \tag{24}$$

of usual unbiased estimator has the model expectation:

$$E_m(V(\bar{y})) = (N-n)\left[\beta^2\theta + \{\delta\Gamma(\theta+g)/\Gamma\theta\}\right]/nN \tag{25}$$

The expressions of $E_m\left(M\left(\bar{y}_a\right)\right)$ and $E_m(V(\bar{y}))$ are not easy task to compare algebraically. Therefore in order to facilitate the comparison, denoting $E_1 = 100E_m(V(\bar{y}))/E_m\left(M\left(\bar{y}_a\right)\right)$ and $E_2 = 100E_m\left(V\left(\bar{y}_r\right)\right)/E_m\left(M\left(\bar{y}_a\right)\right)$, we present below in tables 1, 2, 3, the values of the relative efficiencies of $\bar{y}_a$ with respect to $\bar{y}$ and $\bar{y}_r$ for a few combination of the parametric values under the model (8). Values are given for $N = 60$, $\delta = 2.0$, $\theta = 8$, $\alpha = 0.5, 1.0, 1.5$, $\beta = 0.5, 1.0, 1.5$ and $g = 0.0, 0.5, 1.0, 1.5, 2.0$. The ranges of A, for $\bar{y}_a$ to be better than $\bar{y}_r$ for given $\alpha = 0.5, 1.0, 1.5$ are respectively $(0, 1)$, $(0, 2)$, $(0, 3)$. This clearly indicates that as the size of α increases the range of A for $\bar{y}_a$ to be better than $\bar{y}_r$ increases i.e. flexibility of choosing A increases.

We have made the following observations from the tables 1, 2 and 3:

1. As g increases both E_1 and E_2 decrease. When n increases E_1 increases while E_2 decreases.
2. As α increases (i.e. if the intercept term α departs from origin in positive direction) relative efficiency of $\bar{y}_a$ with respect to $\bar{y}$ decreases while E_2 increases.
3. As β increases E_1 increases for fixed g while E_2 is unaffected.
4. The maximum gain in efficiency is observed over $\bar{y}$ as well as over $\bar{y}_r$ if A coincide with the value of α. Finally, the estimator $\bar{y}_a$ is to be preferred when the intercept term α departs substantially from origin.

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Table 1: Relative efficiencies of $\bar{y}_a$ with respect to $\bar{y}$ and $\bar{y}_r$

$\alpha = 0.5$							
g	β	$n = 10$					
		E_1			E_2		
		A			A		
		0.30	0.60	0.90	0.30	0.60	0.90
0.0	0.5	192.86	193.23	191.40	101.34	101.54	100.57
	1.0	482.16	483.16	478.09	101.34	101.54	100.57
	1.5	964.32	966.17	956.98	101.34	101.54	100.57
0.5	0.5	132.67	132.77	132.30	100.49	100.56	100.21
	1.0	237.82	237.99	237.16	100.49	100.56	100.21
	1.5	413.08	413.36	411.93	100.49	100.56	100.21
1.0	0.5	111.06	111.08	110.95	10.17	100.19	100.07
	1.0	148.08	148.11	147.93	10.17	100.19	100.07
	1.5	209.78	209.83	209.57	10.17	100.19	100.07
1.5	0.5	103.99	104.00	103.96	100.06	100.07	100.03
	1.0	116.64	116.65	116.60	100.06	100.07	100.03
	1.5	137.71	137.72	137.66	100.06	100.07	100.03
2.0	0.5	102.23	102.23	102.22	100.02	100.02	100.01
	1.0	106.43	106.43	106.42	100.02	100.02	100.01
	1.5	113.43	113.43	113.42	100.02	100.02	100.01

$\alpha = 0.5$							
		$n = 20$					
g	β	E_1			E_2		
		A			A		
		0.30	0.60	0.90	0.30	0.60	0.90
0.0	0.5	196.58	196.96	195.11	103.33	101.52	100.56
	1.0	491.46	492.39	487.77	103.33	101.52	100.56
	1.5	982.92	984.39	975.53	103.33	101.52	100.56
0.5	0.5	134.37	134.46	134.46	100.48	100.55	100.20
	1.0	240.86	241.02	240.02	100.48	100.55	100.20
	1.5	418.35	418.63	417.20	100.48	100.55	100.20
1.0	0.5	111.76	111.79	111.65	100.17	100.19	100.07
	1.0	149.01	149.05	148.87	100.17	100.19	100.07
	1.5	211.10	211.16	210.90	100.17	100.19	100.07
1.5	0.5	104.00	104.00	103.96	100.06	100.07	100.02
	1.0	116.64	116.65	116.60	100.06	100.07	100.02
	1.5	137.71	137.73	137.67	100.06	100.07	100.02
2.0	0.5	101.60	101.60	101.58	100.02	100.02	100.01
	1.0	105.77	105.77	105.76	100.02	100.02	100.01
	1.5	112.73	112.73	112.73	100.02	100.02	100.01

Table 2: Relative efficiencies of $\bar{y}_a$ with respect to $\bar{y}$ and $\bar{y}_r$

		$\alpha = 1.0$							
		$n = 10$							
g	β	E_1				E_2			
		A				A			
		0.50	0.10	1.50	1.90	0.50	0.10	1.50	1.90
0.0	0.5	190.31	193.36	190.31	183.82	104.73	106.41	104.73	101.16
	1.0	475.78	483.40	475.78	459.55	104.73	106.41	104.73	101.16
	1.5	951.55	966.79	951.55	919.10	104.73	106.41	104.73	101.16
0.5	0.5	132.03	132.80	132.03	30.34	101.73	102.32	101.73	100.43
	1.0	236.67	238.05	236.67	233.65	101.73	102.32	101.73	100.43
	1.5	411.07	413.46	411.07	405.82	101.73	102.32	101.73	100.43
1.0	0.5	110.87	111.09	110.87	110.36	100.61	100.82	100.61	100.15
	1.0	147.82	148.12	147.82	147.15	100.61	100.82	100.61	100.15
	1.5	209.42	209.84	209.42	208.46	100.61	100.82	100.61	100.15
1.5	0.5	103.93	104.00	103.93	103.77	100.21	100.28	100.21	100.05
	1.0	116.57	116.65	116.57	116.39	100.21	100.28	100.21	100.05
	1.5	137.63	137.73	137.63	137.41	100.21	100.28	100.21	100.05
2.0	0.5	102.21	102.23	102.21	102.15	100.67	100.09	100.07	100.01
	1.0	106.41	106.43	106.41	106.30	100.67	100.09	100.07	100.01
	1.5	113.41	113.43	113.41	113.35	100.67	100.09	100.07	100.01

$\alpha = 1.0$									
		$n = 20$							
g	β	E_1				E_2			
		A				A			
		0.50	0.10	1.50	1.90	0.50	0.10	1.50	1.90
0.0	0.5	194.01	197.08	194.01	187.47	104.67	106.33	104.67	101.14
	1.0	485.03	492.70	485.03	468.68	104.67	106.33	104.67	101.14
	1.5	970.06	985.40	970.06	937.36	104.67	106.33	104.67	101.14
0.5	0.5	133.73	134.49	133.73	132.05	101.70	102.28	101.70	100.08
	1.0	239.71	241.08	239.71	236.71	101.70	102.28	101.70	100.08
	1.5	416.35	418.73	416.35	411.13	101.70	102.28	101.70	100.08
1.0	0.5	111.07	111.08	111.07	111.08	100.60	100.80	100.60	100.15
	1.0	148.77	149.06	148.77	148.11	100.60	100.80	100.60	100.15
	1.5	210.75	211.17	210.75	209.82	100.60	100.80	100.60	100.15
1.5	0.5	103.94	104.01	103.94	103.78	100.20	100.27	100.20	100.05
	1.0	116.57	116.65	116.57	116.40	100.20	100.27	100.20	100.05
	1.5	137.64	137.73	137.64	137.42	100.20	100.27	100.20	100.05
2.0	0.5	101.58	101.60	101.58	101.52	100.07	100.09	100.07	100.01
	1.0	105.75	105.77	105.75	105.70	100.07	100.09	100.07	100.01
	1.5	112.71	112.73	112.71	112.65	100.07	100.09	100.07	100.01

Table 3: Relative efficiencies of $\bar{y}_a$ with respect to $\bar{y}$ and $\bar{y}_r$

$\alpha = 1.5$											
g	β	$n = 10$									
		E_1					E_2				
		A					A				
		0.60	1.20	1.80	2.40	2.90	0.60	1.20	1.80	2.40	2.90
0.0	0.5	183.82	192.25	192.25	183.82	171.79	108.77	113.76	113.76	108.77	101.65
	1.0	459.55	480.62	480.62	459.55	429.47	108.77	113.76	113.76	108.77	101.65
	1.5	919.10	961.25	961.25	919.10	858.94	108.77	113.76	113.76	108.77	101.65
0.5	0.5	130.34	132.52	132.52	130.34	127.01	103.29	105.01	105.01	103.29	100.64
	1.0	233.64	237.55	237.55	233.65	227.67	103.29	105.01	105.01	103.29	100.64
	1.5	405.82	412.60	412.60	405.82	395.44	103.29	105.01	105.01	103.29	100.64
1.0	0.5	110.36	111.01	111.01	110.36	109.34	101.17	101.77	101.77	101.17	100.23
	1.0	147.15	148.02	148.02	147.15	147.79	101.17	101.77	101.77	101.17	100.23
	1.5	208.46	209.69	209.69	208.46	206.53	101.17	101.77	101.77	101.17	100.23
1.5	0.5	103.77	103.98	103.98	103.77	103.44	100.40	100.60	100.60	100.40	100.08
	1.0	116.39	116.62	116.62	116.39	116.01	100.40	100.60	100.60	100.40	100.08
	1.5	137.41	137.69	137.69	137.41	139.68	100.40	100.60	100.60	100.40	100.08
2.0	0.5	102.15	102.22	102.22	102.15	102.04	100.13	100.20	100.20	100.13	100.03
	1.0	106.35	106.42	106.42	106.35	106.24	100.13	100.20	100.20	100.13	100.03
	1.5	113.35	113.42	113.42	113.35	113.23	100.13	100.20	100.20	100.13	100.03

		$\alpha = 1.5$									
		$n = 20$									
g	β	E_1					E_2				
		A					A				
		0.60	1.20	1.80	2.40	2.90	0.60	1.20	1.80	2.40	2.90
0.0	0.5	187.47	196.97	195.97	187.47	175.33	108.67	113.59	113.59	108.67	101.63
	1.0	468.68	489.91	489.91	468.68	438.34	108.67	113.59	113.59	108.67	101.63
	1.5	937.36	979.83	979.83	937.36	876.67	108.67	113.59	113.59	108.67	101.63
0.5	0.5	132.05	134.21	134.21	132.05	128.73	103.23	104.92	104.92	103.23	100.63
	1.0	236.70	240.58	240.58	236.70	230.76	103.23	104.92	104.92	103.23	100.63
	1.5	411.13	417.87	417.87	411.13	400.80	103.23	104.92	104.92	103.23	100.63
1.0	0.5	111.08	111.72	111.72	111.08	110.08	101.14	101.72	101.72	101.14	100.23
	1.0	148.11	148.96	148.96	148.11	146.77	101.14	101.72	101.72	101.14	100.23
	1.5	209.82	211.02	211.02	209.82	207.92	101.14	101.72	101.72	101.14	100.23
1.5	0.5	103.78	103.98	103.98	103.78	103.46	100.39	100.58	100.58	100.39	100.08
	1.0	116.40	116.62	116.62	116.40	116.40	100.39	100.58	100.58	100.39	100.08
	1.5	137.43	137.70	137.70	137.43	137.00	100.39	100.58	100.58	100.39	100.08
2.0	0.5	101.53	101.59	101.59	101.53	101.42	100.13	100.19	100.19	100.03	100.03
	1.0	105.70	105.77	105.77	105.70	105.59	100.13	100.19	100.19	100.03	100.03
	1.5	112.65	112.72	112.72	112.65	112.54	100.13	100.19	100.19	100.03	100.03